Differential Motions and Velocities

Jacobian matrix

- Multi input & multi output system
  - System Model

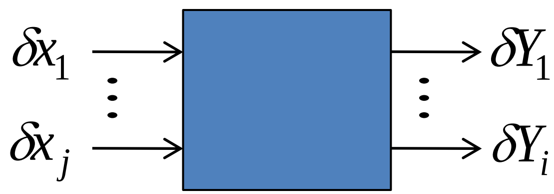
$$Y_1 = f_1(x_1, \cdots, x_j)$$

$$\vdots$$

$$Y_i = f_i(x_1, \cdots, x_j)$$

Jacobian Matrix

- Differential System
  - Chain rule



$$\delta Y_1 = \frac{\partial f_1}{\partial x_1}\delta x_1 + \cdots + \frac{\partial f_1}{\partial x_j}\delta x_j$$

$$\vdots$$

$$\delta Y_i = \frac{\partial f_i}{\partial x_1}\delta x_1 + \cdots + \frac{\partial f_i}{\partial x_j}\delta x_j$$

- Matrix Form

$$\begin{bmatrix} \delta Y_1 \\ \vdots \\ \delta Y_i \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_j} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_i}{\partial x_1} & \cdots & \frac{\partial f_i}{\partial x_j} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \vdots \\ \delta x_j \end{bmatrix}$$

J : Jacobian matrix

Differential Motion

- Forward Kinematics

$$x = f_1(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$$
$$y = f_2(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$$
$$z = f_3(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$$
$$\phi_x = f_4(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$$
$$\phi_y = f_5(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$$
$$\phi_z = f_6(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$$

- Differential Motion

$$\begin{bmatrix} dx \\ dy \\ dz \\ d\phi_x \\ d\phi_y \\ d\phi_z \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \cdots & \frac{\partial f_1}{\partial \theta_6} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_6}{\partial \theta_1} & \cdots & \frac{\partial f_6}{\partial \theta_6} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \\ d\theta_5 \\ d\theta_6 \end{bmatrix} \quad \Leftrightarrow \quad D = J\ D_\theta$$

Differential Motion

- Velocities

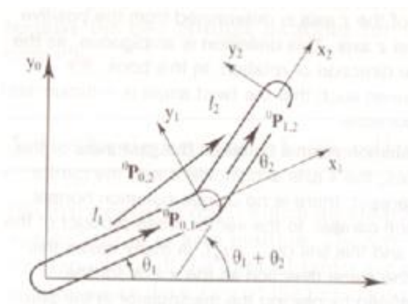
$$\begin{bmatrix} dx/dt \\ dy/dt \\ dz/dt \\ d\phi_x/dt \\ d\phi_y/dt \\ d\phi_z/dt \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \cdots & \frac{\partial f_1}{\partial \theta_6} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_6}{\partial \theta_1} & \cdots & \frac{\partial f_6}{\partial \theta_6} \end{bmatrix} \begin{bmatrix} d\theta_1/dt \\ d\theta_2/dt \\ d\theta_3/dt \\ d\theta_4/dt \\ d\theta_5/dt \\ d\theta_6/dt \end{bmatrix}$$

$\Leftrightarrow$ $$V = J(\Theta)\dot{\Theta}$$

$\Leftrightarrow$ $$\dot{\Theta} = J^{-1}(\Theta)V$$

Examples

(ex1) Type 1 robot

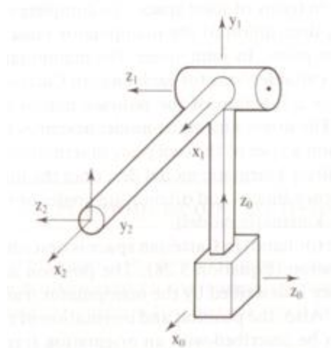


$$x = l_1 c1 + l_2 c12$$
$$y = l_1 s1 + l_2 s12$$

$l_1 = l_2 = 1(m)$, $\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{6}, \dot{\theta}_1 = 2$(rad/s), $\dot{\theta}_2 = 4$(rad/s) => $\dot{x}, \dot{y}$ = ?

Examples

(ex1) Type 4 robot



$$
{}^{R}T_{H} = \begin{bmatrix} c2 & -s2 & 0 & l_2 c_2 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & d_1 + l_2 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

=> Jacobian matrix ?

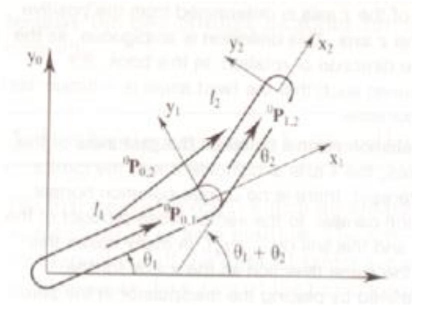
Singularities

- Singularities (특이점)
  - Inverse Jacobian ($J^{-1}(\Theta)$) 이 존재하지 않는 joint 위치 (Θ)
    - $\Rightarrow$ $\det(J(\Theta)) = 0$ 인 Θ
    - $\Rightarrow$ Joint 구동 시 joint velocity $= \infty$ 인 joint 위치
  - Type
    - Workspace boundary singularities
    - Workspace interior singularities

(ex) Type 1 robot

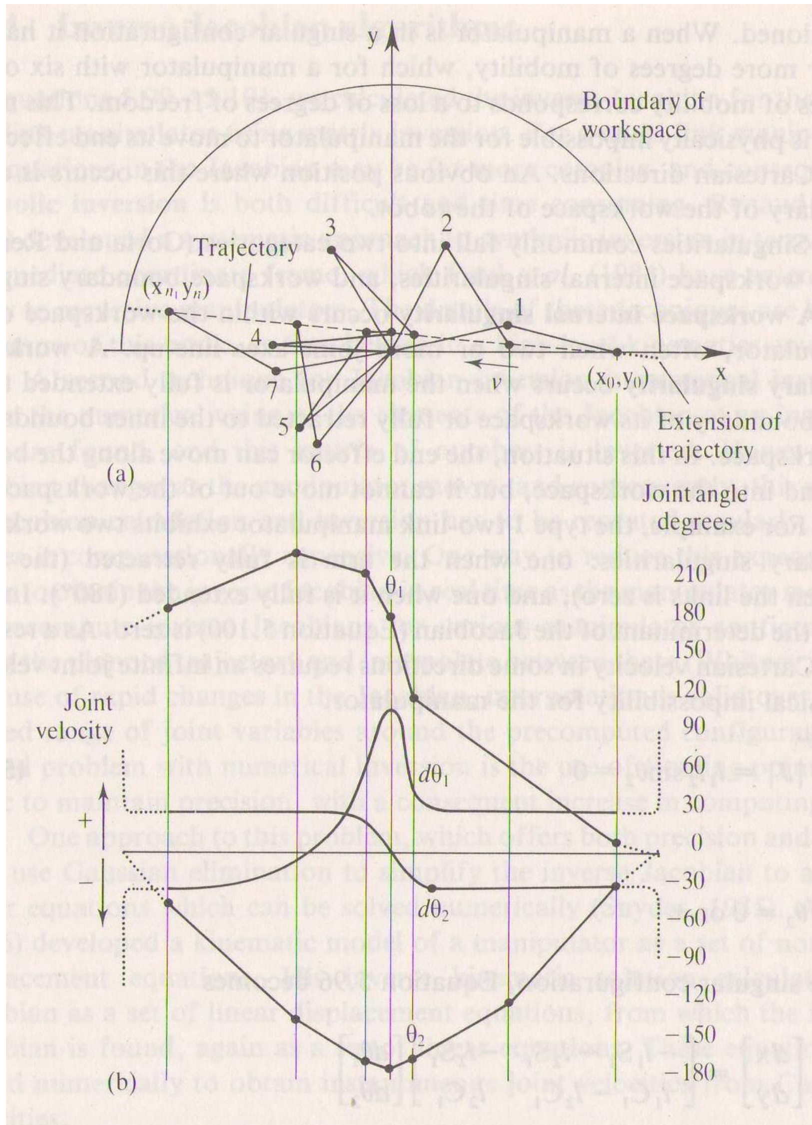
Singularities

(ex) Type 1 robot



$$J = \begin{bmatrix} -l_1 s1 - l_2 s12 & -l_2 s12 \\ l_1 c1 + l_2 c12 & l_2 c12 \end{bmatrix}$$

Singularities



Singularities

- Singularities Avoidance
  - 특이점 회피 이동경로 계획