Ant Colony Optimization

Swarm Intelligence (SI)

- Collective behavior of decentralized, self-organized systems. (G.Beni and J.Wang (1989, cellular robotics)

Swarm Intelligence

- Collective system capable of accomplishing difficult tasks in dynamic and varied environments without any external guidance or control and with no central coordination

- Achieving a collective performance which could not normally be achieved by an individual acting alone

(ex)

Ant colony optimization (ACO) algorithm,
Artificial bee colony (ABC) algorithm,
Artificial immune systems (AIS),
Particle swarm optimization (PSO),
....

Ant Colony Optimization

- A probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs.

- A member of swarm intelligence methods, and it constitutes some metaheuristic optimization

- Initially proposed by Marco Dorico in 1992 in his PhD thesis
  - Aims to search for an optimal path in a graph, based on the behavior of ants seeking a path between their colony and a source of food.

Ant Colony Optimization

Problem name	Authors	Algorithm name	Year
Traveling salesman	Dorigo, Maniezzo & Colorni	AS	1991
	Gamberdella & Dorigo	Ant-Q	1995
	Dorigo & Gamberdella	ACS &ACS 3 opt	1996
	Stutzle & Hoos	MMAS	1997
	Bullnheimer, Hartl & Strauss	AS_{rank}	1997
	Cordon, et al.	BWAS	2000
Quadratic assignment	Maniezzo, Colorni & Dorigo	AS-QAP	1994
	Gamberdella, Taillard & Dorigo	HAS-QAP	1997
	Stutzle & Hoos	MMAS-QAP	1998
	Maniezzo	ANTS-QAP	1999
	Maniezzo & Colorni	AS-QAP	1994
Scheduling problems	Colorni, Dorigo & Maniezzo	AS-JSP	1997
	Stutzle	AS-SMTTP	1999
	Barker et al	ACS-SMTTP	1999
	den Besten, Stutzle & Dorigo	ACS-SMTWTP	2000
	Merkle, Middenderf & Schmeck	ACO-RCPS	1997
Vehicle routing	Bullnheimer, Hartl & Strauss	AS-VRP	1999
	Gamberdella, Taillard & Agazzi	HAS-VRP	1999

Ant Colony Optimization

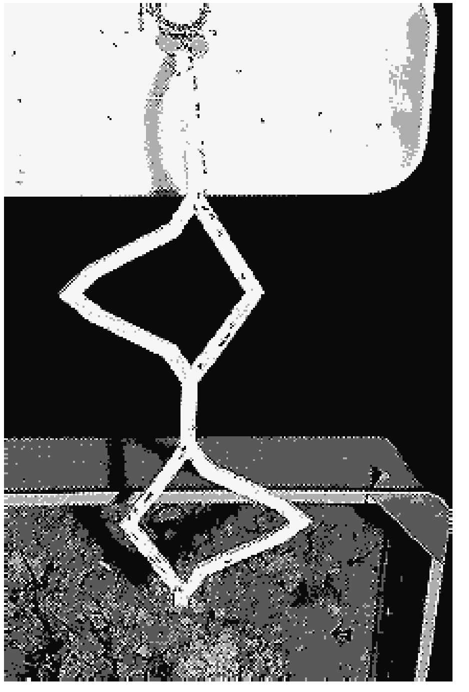
Problem name	Authors	Algorithm name	Year
Connection-oriented network routing	Schoonderwood et al.	ABC	1996
	White, Pagurek & Oppacher	ASGA	1998
	Di Caro & Dorigo	AntNet-FS	1998
	Bonabeau et al.	ABC-smart ants	1998
Connection-less network routing	Di Caro & Dorigo	AntNet & AntNet-FA	1997
	Subramanian, Druschel & Chen	Regular ants	1997
	Heusse et al.	CAF	1998
	van der Put & Rethkrantz	ABC-backward	1998
Sequential ordering	Gamberdella& Dorigo	HAS-SOP	1997
Graph coloring	Costa & Hertz	ANTCOL	1997
Shortest common supersequence	Michel & Middendorf	AS_SCS	1998
Frequency assignment	Maniezzo & Carbonaro	ANTS-FAP	1998
Generalized assignment	Ramalhinho Lourenco & Serra	MMAS-GAP	1998
Multiple knapsack	Leguizamon & Michalewicz	AS-MKP	1999
Optical networks routing	Navarro Varela & Sinclair	ACO-VWP	1999
Redundancy allocation	Liang & Smith	ACO-RAP	1999
Constraint satisfaction	Solnon	Ant-P-solver	2000

Ant’s Foraging Behavior

- Experiments with Argentine ants

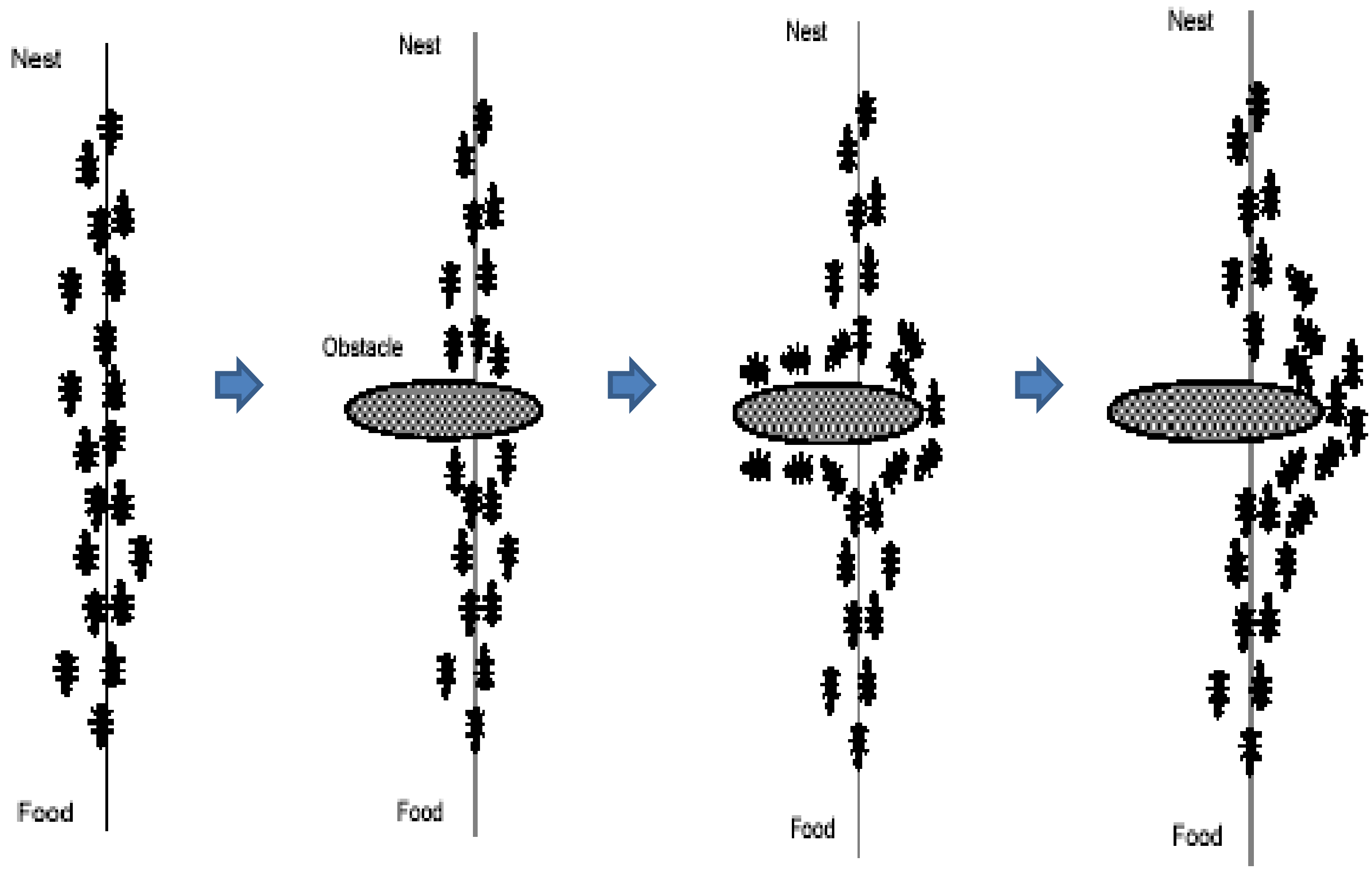
  (Goss et.al, 1989)
  - Ants go from the nest to the food source and backwards
  - after a while, the ants prefer the shortest path from the nest to the food source
  - stigmercy:
    - the ants communicate indirectly laying pheromone trails and following trails with higher pheromone
    - length gradient → pheromone will accumulate on the shortest path

food



nest

Ant’s Foraging Behavior



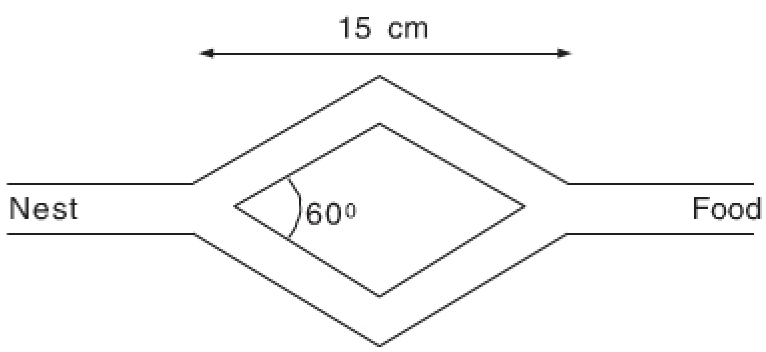
Double Bridge Experiments

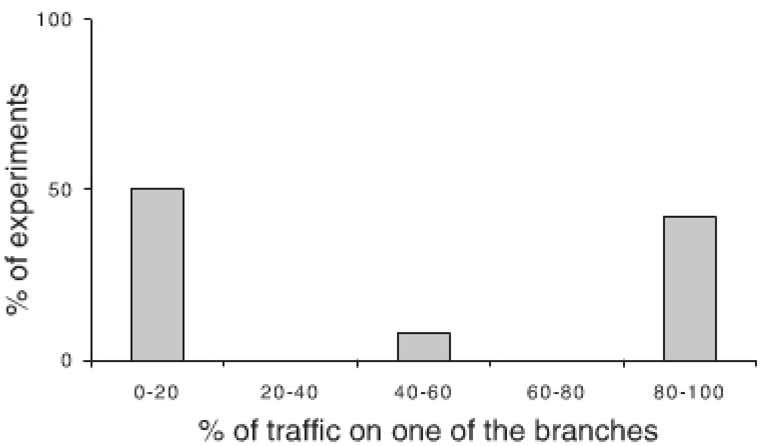
- Deneubourg et. al., 1990

1) Branches of equal length
   - $r = {}^{l_l}/_{l_s} = 1$
   - ants converges to one single path

➢ Initial random fluctuation 의 효과

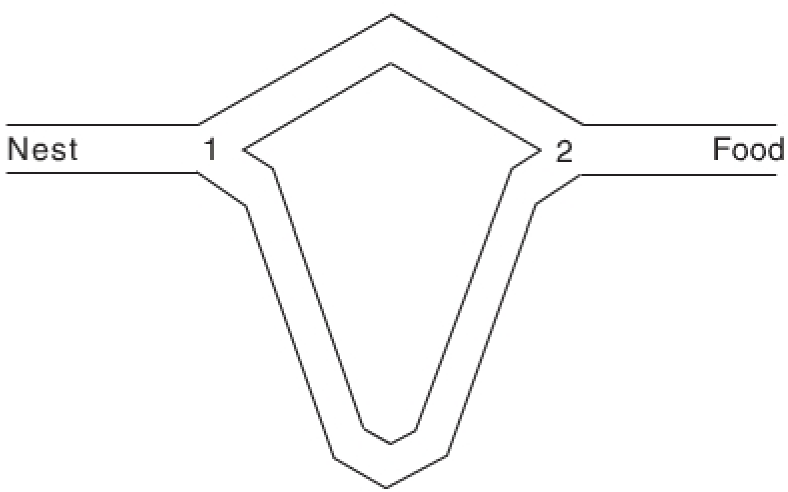
➢ Autocatalytic or positive feedback process

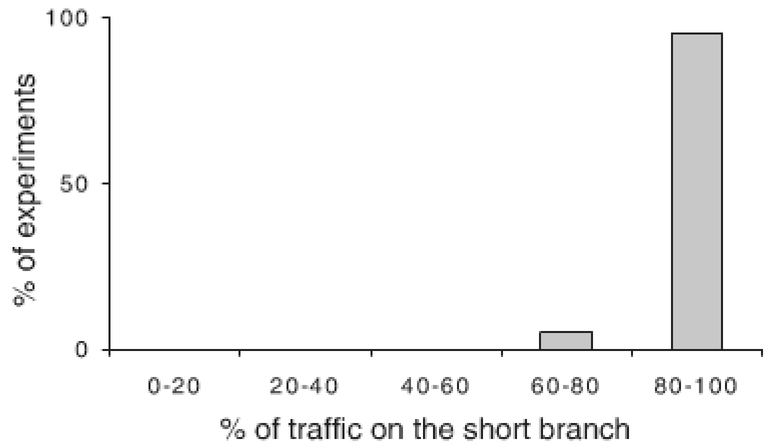




Double Bridge Experiments

2) Branches of different length (1)
   - $r = {}^{l_l}/_{l_s} = 2$
   - ants converges to the short branch
     - Initial random fluctuation 의 효과가 줄어듬
     - Positive feedback to shorter branch
       - Shorter branch -> Higher level of pheromone -> faster accumulation of pheromone on shorter branch -> ….

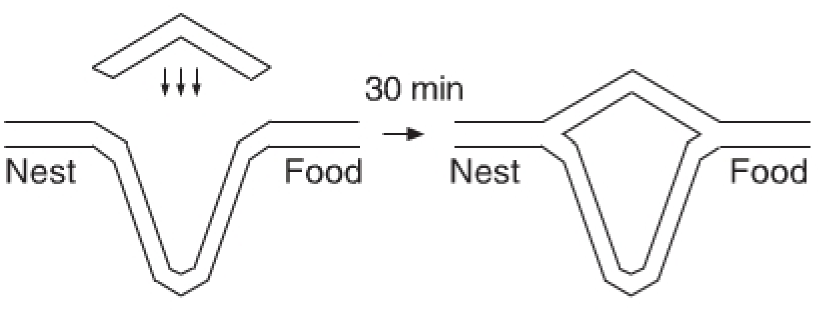


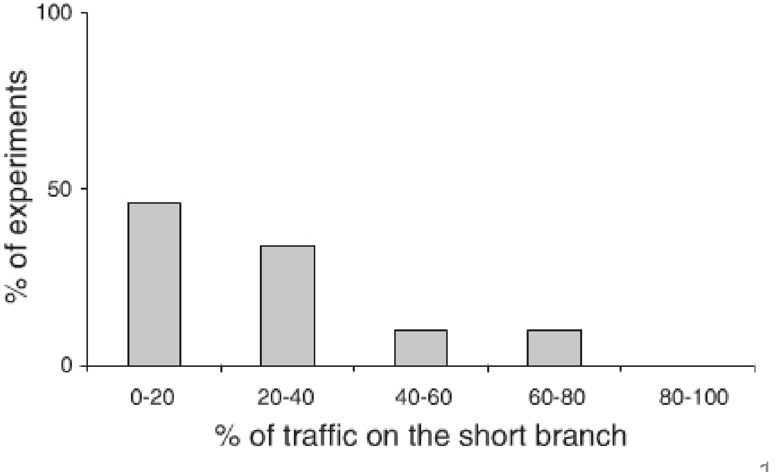


Double Bridge Experiments

3) Branches of different length (2)
   - $r = {}^{l_l}/_{l_s} = 2$
   - Short branch were added after 30 min
   - the great majority of ants select the long branch
     - High pheromone concentration
     - Slow evaporation of pheromone

➢ Autocatalytic (自動觸媒) behavior





Double Bridge Experiments

- Stochastic Model
  - Probability of choosing a branch

$$p_{is}(t) = \frac{(t_s + \varphi_{is}(t))^\alpha}{(t_s + \varphi_{is}(t))^\alpha + (t_s + \varphi_{il}(t))^\alpha}$$

$$p_{is}(t) + p_{il}(t) = 1$$

$p_{ia}(t)$: probability that an ant arriving at decision point $i \in \{0,1\}$ selects branch $a \in \{s, l\}$ (s: short, l: long)

$\varphi_{ia}(t)$: total amount of pheromone at instant t

t_s: traverse time on short branch
$r\ t_s$: traverse time on long branch

$\alpha = 2$

- Pheromone evaporation

$$d\varphi_{is}/dt = \psi p_{js}(t - t_s) + \psi p_{is}(t), \qquad (i = 1, j = 2; i = 2, j = 1),$$

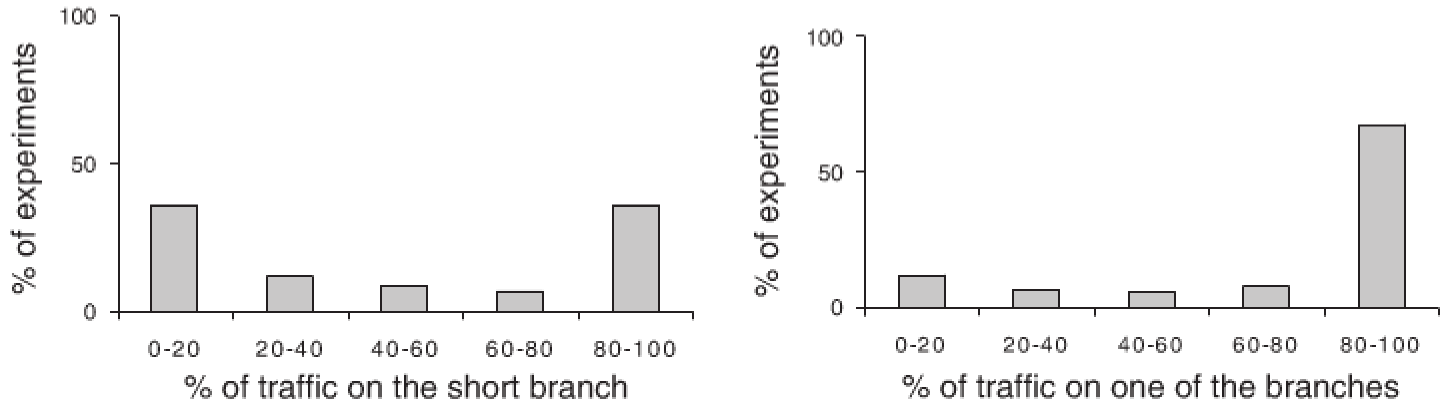
$$d\varphi_{il}/dt = \psi p_{jl}(t - r \cdot t_s) + \psi p_{il}(t), \qquad (i = 1, j = 2; i = 2, j = 1)$$

ψ: ants per second cross the bridge in each direction

Double Bridge Experiments

- Monte Carlo simulation

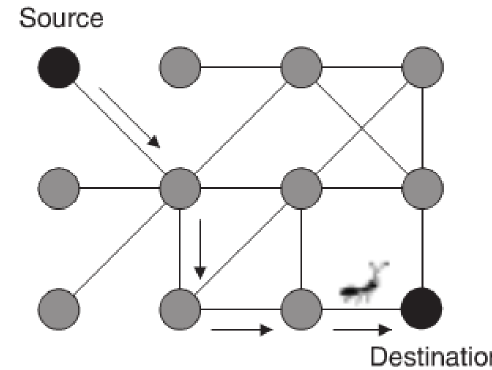
$r = 1$ $\qquad\qquad$ $r = 2$



(a) $\qquad\qquad$ (b)

S-ACO Algorithm

Graph $G = (N, A)$, N: set of nodes, A:set of arcs



τ_{ij}: pheromone trail on arc (i, j)

= const. , $\forall (i, j) \in A$ (at the beginning stage)

- Ant's path-searching behavior
  - If an ant k is located at a node i, then the probability of choosing j as next node is

$$p_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^{\alpha}}{\sum_{l \in \mathcal{N}_i^k} \tau_{il}^{\alpha}}, & \text{if } j \in \mathcal{N}_i^k \\ 0, & \text{if } j \notin \mathcal{N}_i^k \end{cases}$$

N_i^k: neighborhood of ant k when in node i

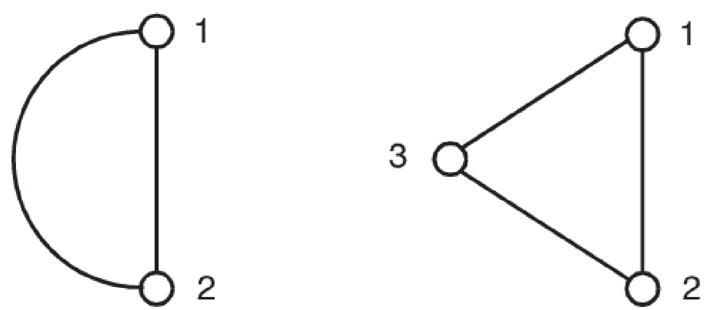
All nodes directly connected to node i except for the predecessor of node i

S-ACO Algorithm

- Pheromone update
  - k-th ant deposits an amount $\Delta\tau^k$ of pheromone on arcs it has visited

$$\tau_{ij} = \tau_{ij} + \Delta\tau^k$$

<Equivalent graph model>



(a) (b)

$\Delta\tau^k$: non-increasing function of path length

$\Delta\tau^k$: constant

S-ACO Algorithm

- Pheromone evaporation
  - After each ant k has moved to a next node, pheromone trails are evaporated by

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} \qquad \forall (i,j) \in A \qquad\qquad \rho \in [0,1]$$

  - Exponentially decreasing

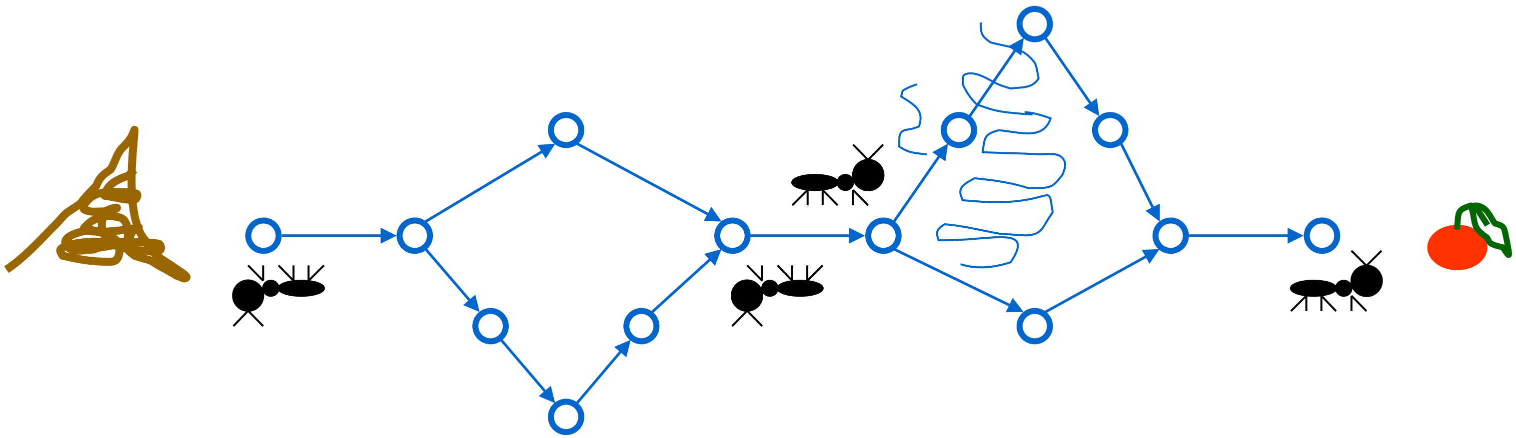
S-ACO

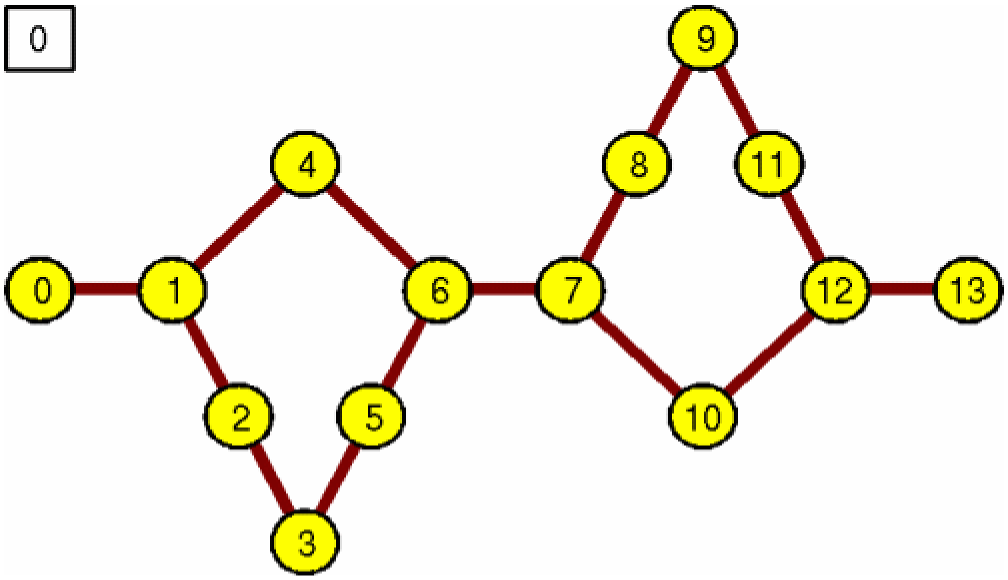
- Shortest Path Algorithm
  - Given a directed acyclic graph $G = (N, A)$
    , where $N = \{0, \cdots, G\}$ 0: source (nest), G: destination (food)

```
for all ant k,
        ant position n^k ← 0
        ant state s^k ← forward
for all arcs (i, j),
        τ_ij ← const

repeat
    for all ant k,
            ant_step(k)
    for all arcs (i, j),
        τ_ij ← (1 − ρ)τ_ij
```

```
ant_step(k)

if n^k = G, s^k ← backward
if n^k = 0, s^k ← forward

if s^k = forward,
    choose next ant position n^k ← j
    with the probability of p^k_ij
    update pheromone τ_ij = τ_ij + ε

if s^k = backward,
    choose next ant position n^k ← j
    with the probability of p^k_ji
    update pheromone τ_ji = τ_ji + ε
```

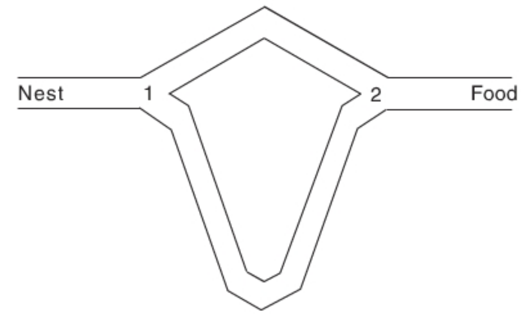
S-ACO





Experiments with Double Bridge

- Number of ants
  $m = 1, \cdots, 512$



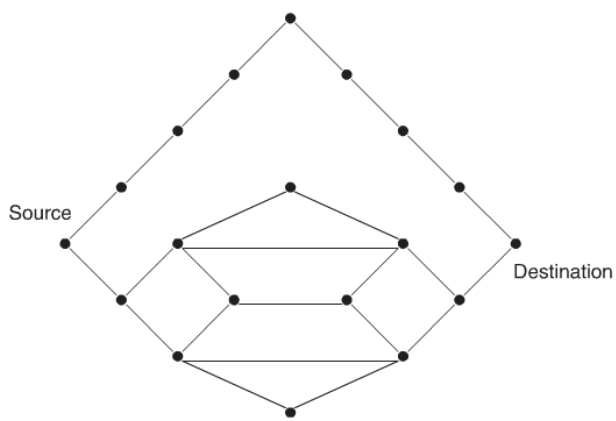
- Pheromone update
  $\Delta\tau^k$ = constant (without considering path length)
  $= {}^{1}/_{L^k}$ (with considering path length)

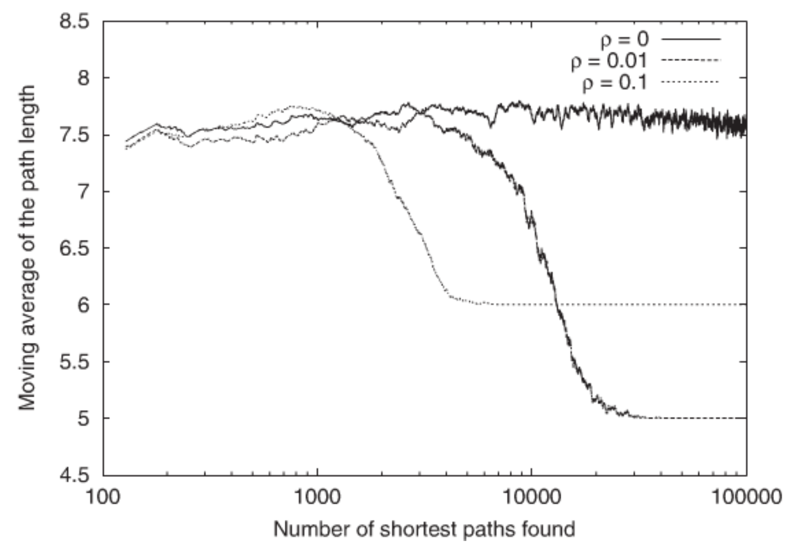
Table 1.1
Percentage of trials in which S-ACO converged to the long path (100 independent trials for varying values of m, with $\alpha = 2$ and $\rho = 0$)

m	1	2	4	8	16	32	64	128	256	512
without path length	50	42	26	29	24	18	3	2	1	0
with path length	18	14	8	0	0	0	0	0	0	0

Experiments with Extended Double Bridge

- Pheromone evaporation
  - Evaporation rate $\rho = \{0, 0.01, 0.1\}$ $\tau_{ij} \leftarrow (1-\rho)\tau_{ij}$ $\forall (i,j) \in A$
    - $\rho = 0$: no evaporation
    - $\rho = 0.01$: 10% of pheromone evaporated after 10 iterations
    - $\rho = 0.1$: 65% of pheromone evaporated after 10 iterations
  - The larger ρ, the faster converges to suboptimal solution





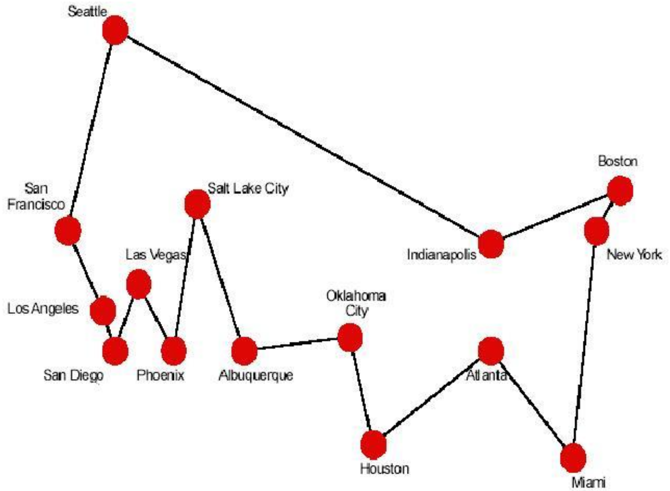
Traveling Salesman Problem

- TSP PROBLEM

  Given N cities, and a distance function d between cities, find a tour that:

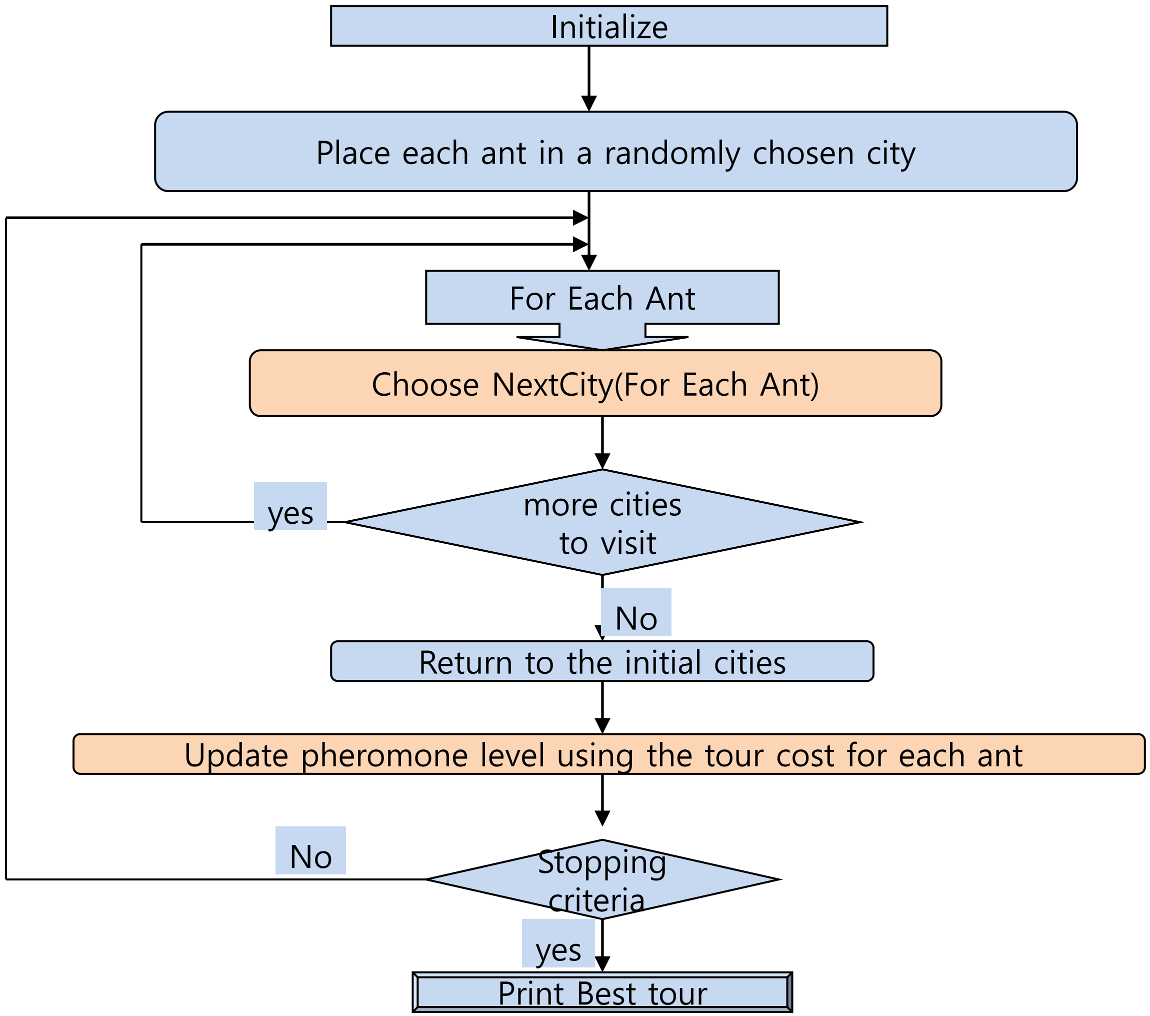
  1. Goes through every city once and only once
  2. Minimizes the total distance.

  – Problem is NP-hard
  – Classical combinatorial optimization problem to test.



Ant System (AS) for TSP

- Algorithm



Ant System (AS) for TSP

- Tour Construction
  - Probability of ant k, currently at city i, choosing a next city j

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k} [\tau_{ik}(t)]^{\alpha}[\eta_{il}]^{\beta}} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases}$$

$N^k{}_i$: feasible neighborhood of ant k when being at city i
set of cities that ant k has not visited yet

$\eta_{ij} = {}^{1}/_{d_{ij}}$: heuristic value priori to closer city

d_{ij}: length of arc (i, j)

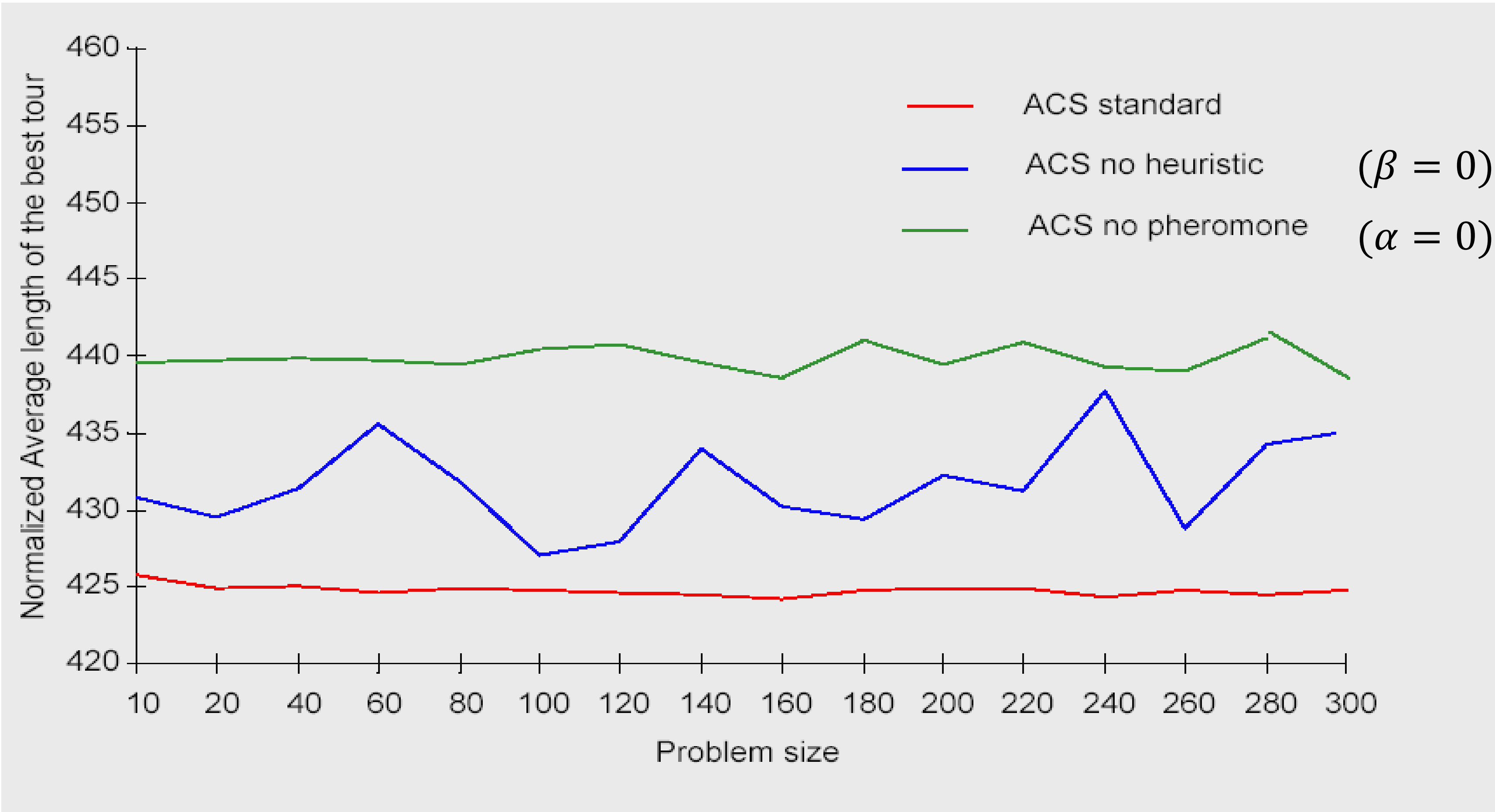
α, β : parameters

$\alpha = 0, \beta > 1$: the closest city is selected

$\alpha > 1, \beta = 0$: only pheromone is used

$\Rightarrow$ stagnation

$\Rightarrow$ strongly suboptimal solution


Normalized Average length of the best tour
460
455
450
445
440
435
430
425
420
10
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
Problem size
ACS standard
ACS no heuristic
ACS no pheromone
$(\beta = 0)$
$(\alpha = 0)$

Ant System (AS) for TSP

- Pheromone trail update

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau^{k}{}_{ij} \qquad , \forall (i,j) \in A$$

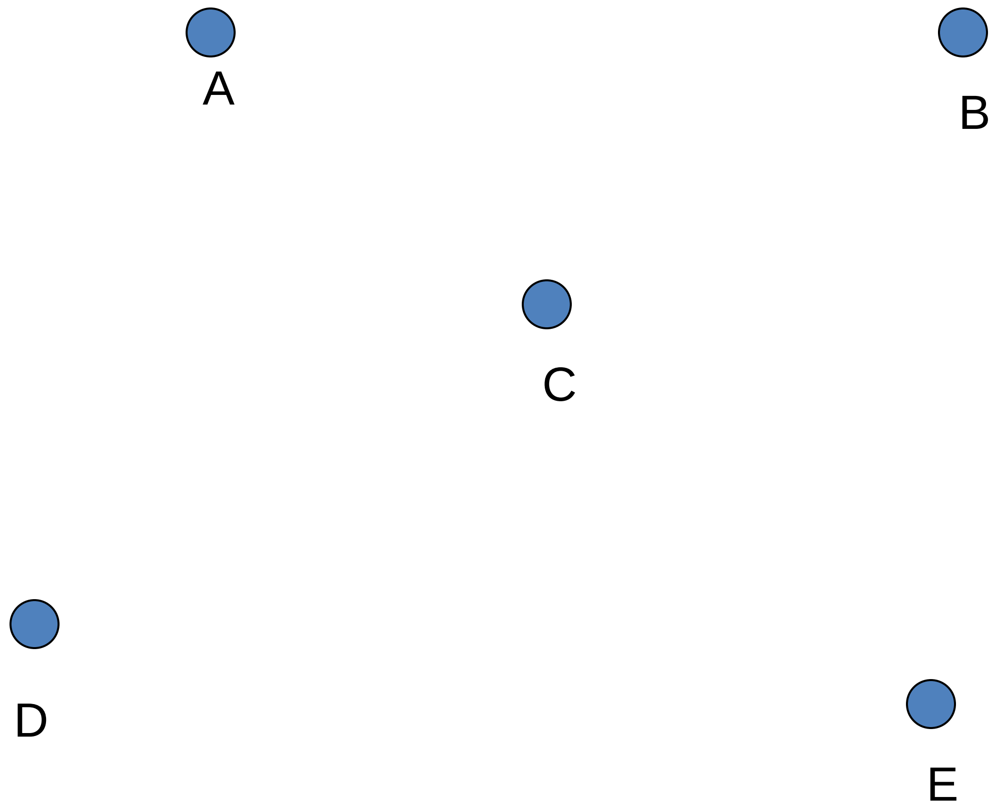
$$\Delta\tau^{k}{}_{ij} = \begin{cases} 1/C^{k}, & \text{if } (i,j) \in T^{k} \\ 0, & \text{otherwise} \end{cases}$$

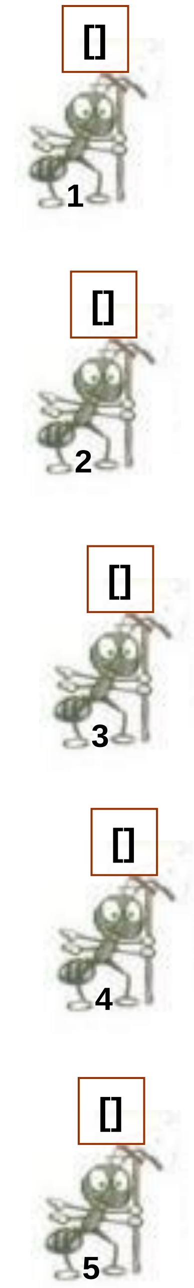
T^k : tour of ant k, tabu

- Pheromone evaporation

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} \qquad \forall (i,j) \in A$$

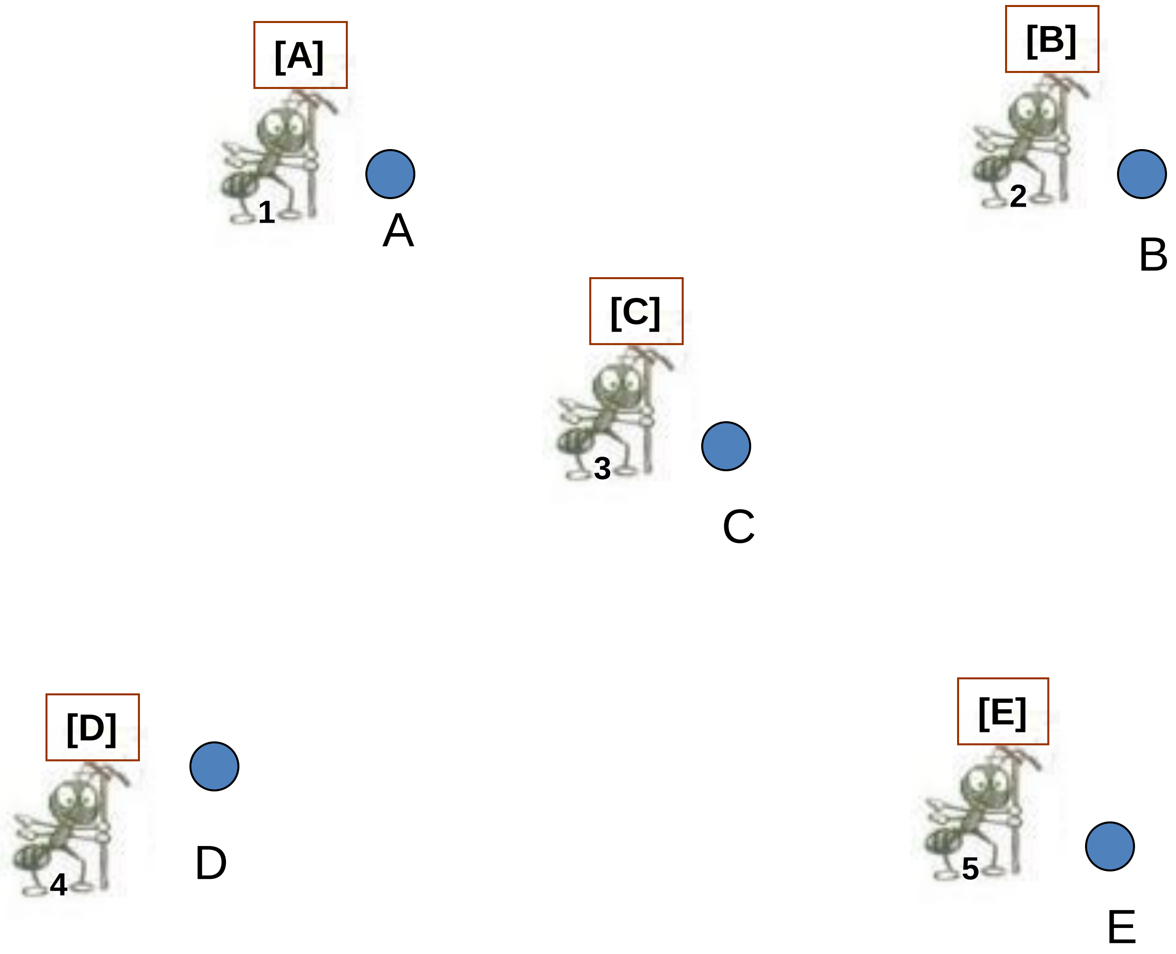
A simple TSP example



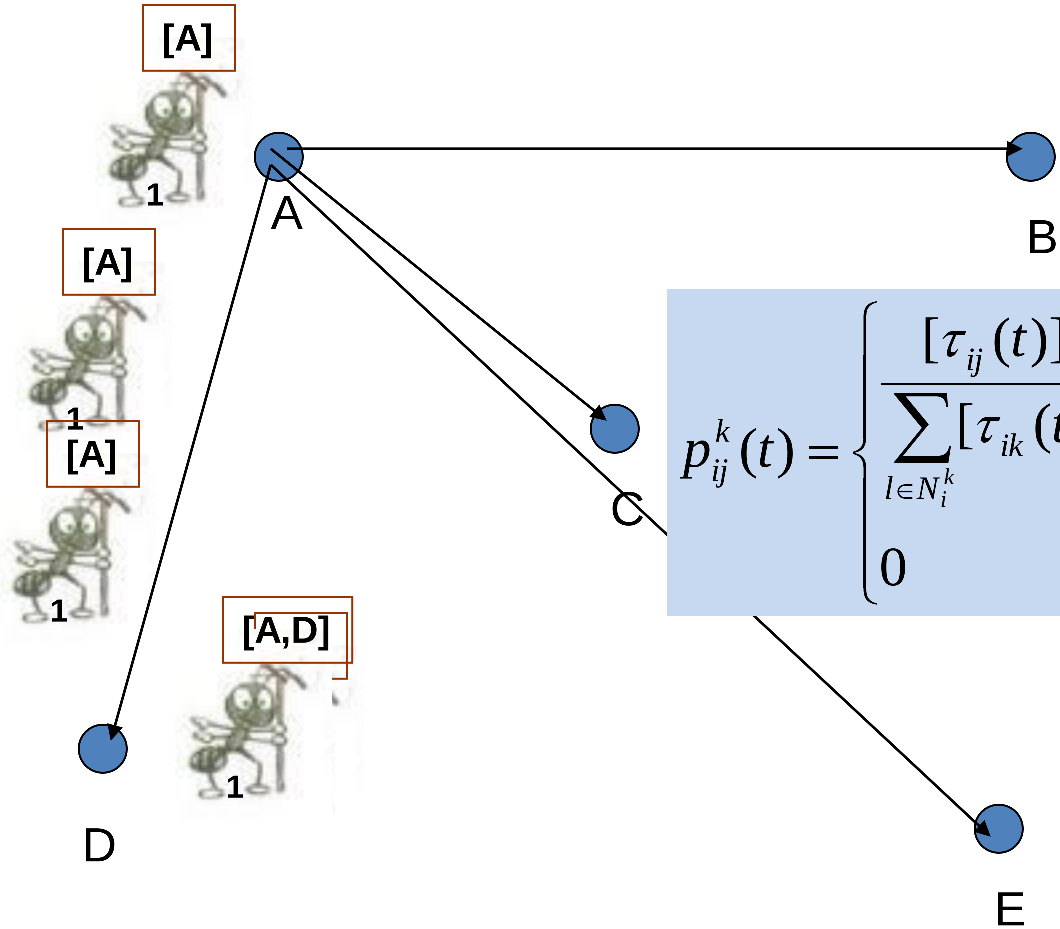


d_{AB} =100;d_{BC} = 60…;d_{DE} =150

Iteration 1

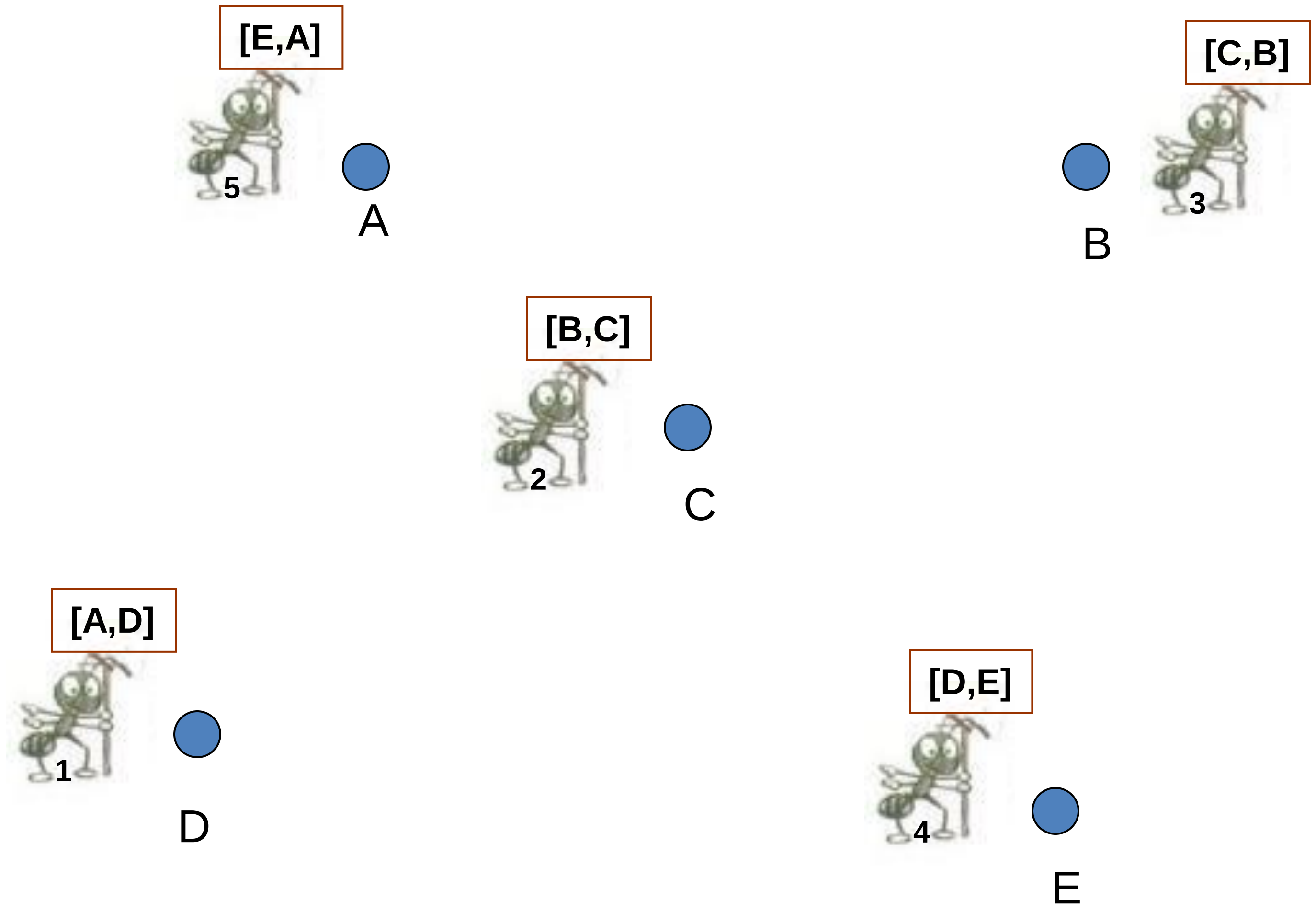


How to build next sub-solution?

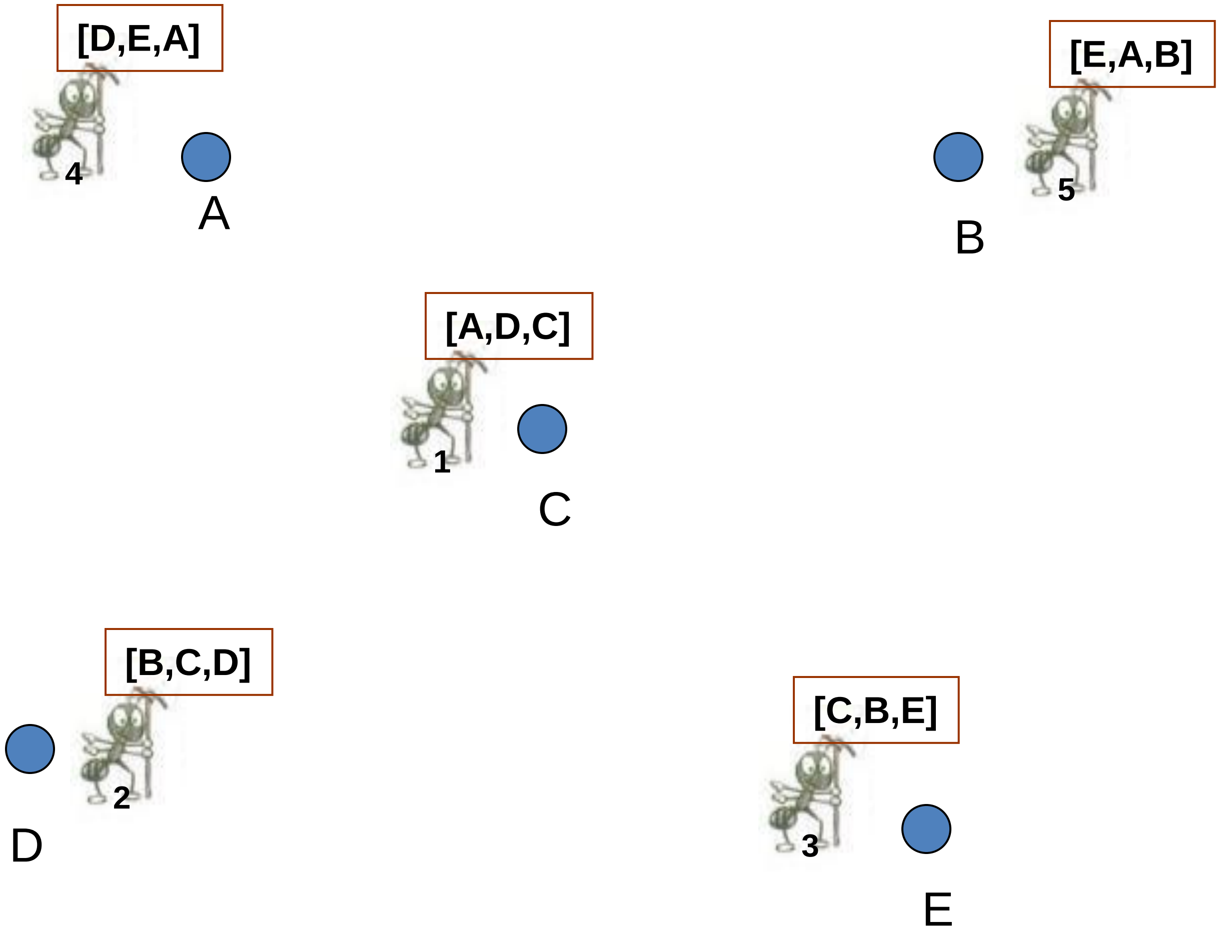


$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k} [\tau_{ik}(t)]^{\alpha}[\eta_{il}]^{\beta}} & \text{if } \ j \in N_i^k \\ 0 & \text{if } \ j \notin N_i^k \end{cases}$$

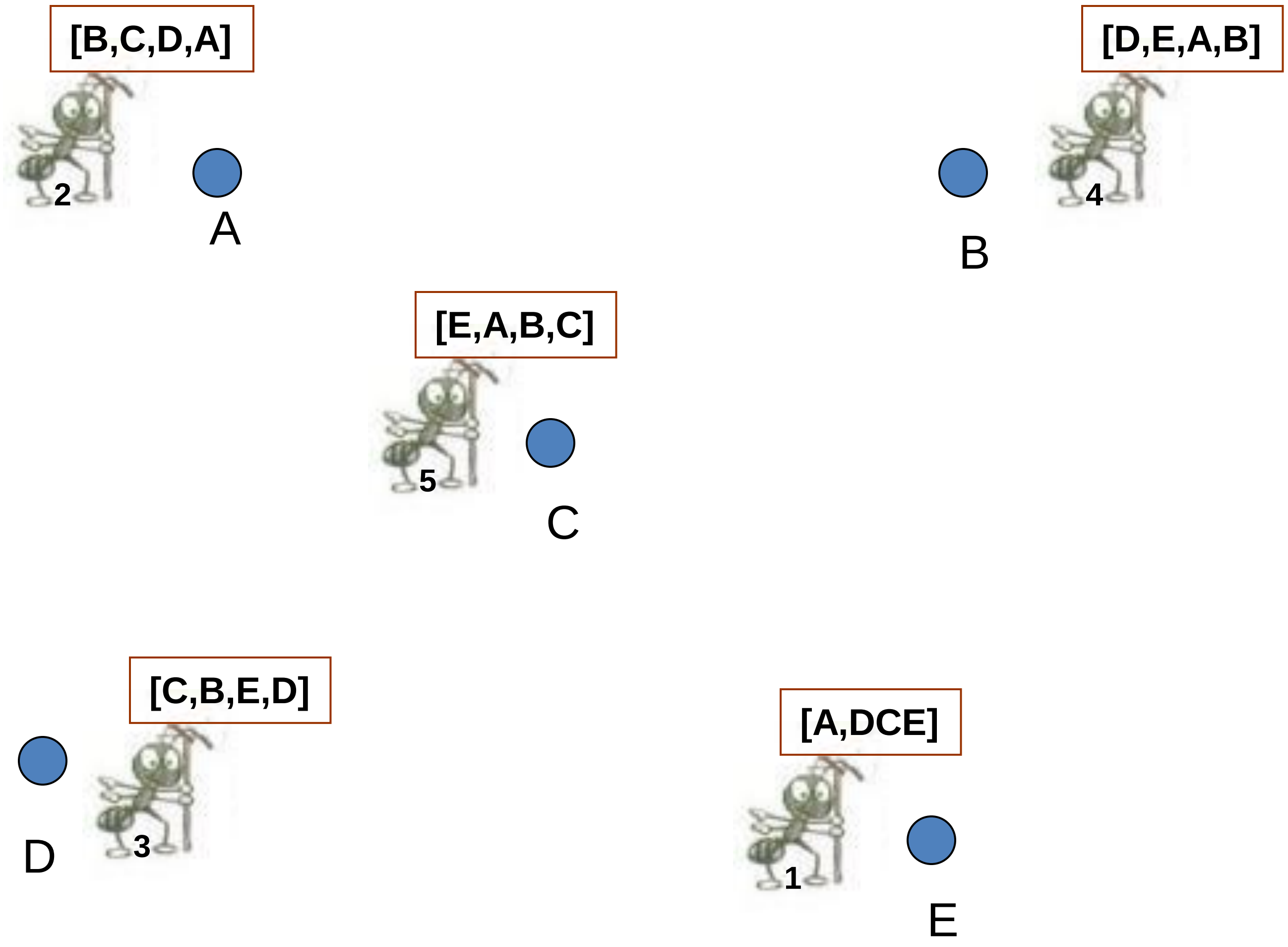
Iteration 2



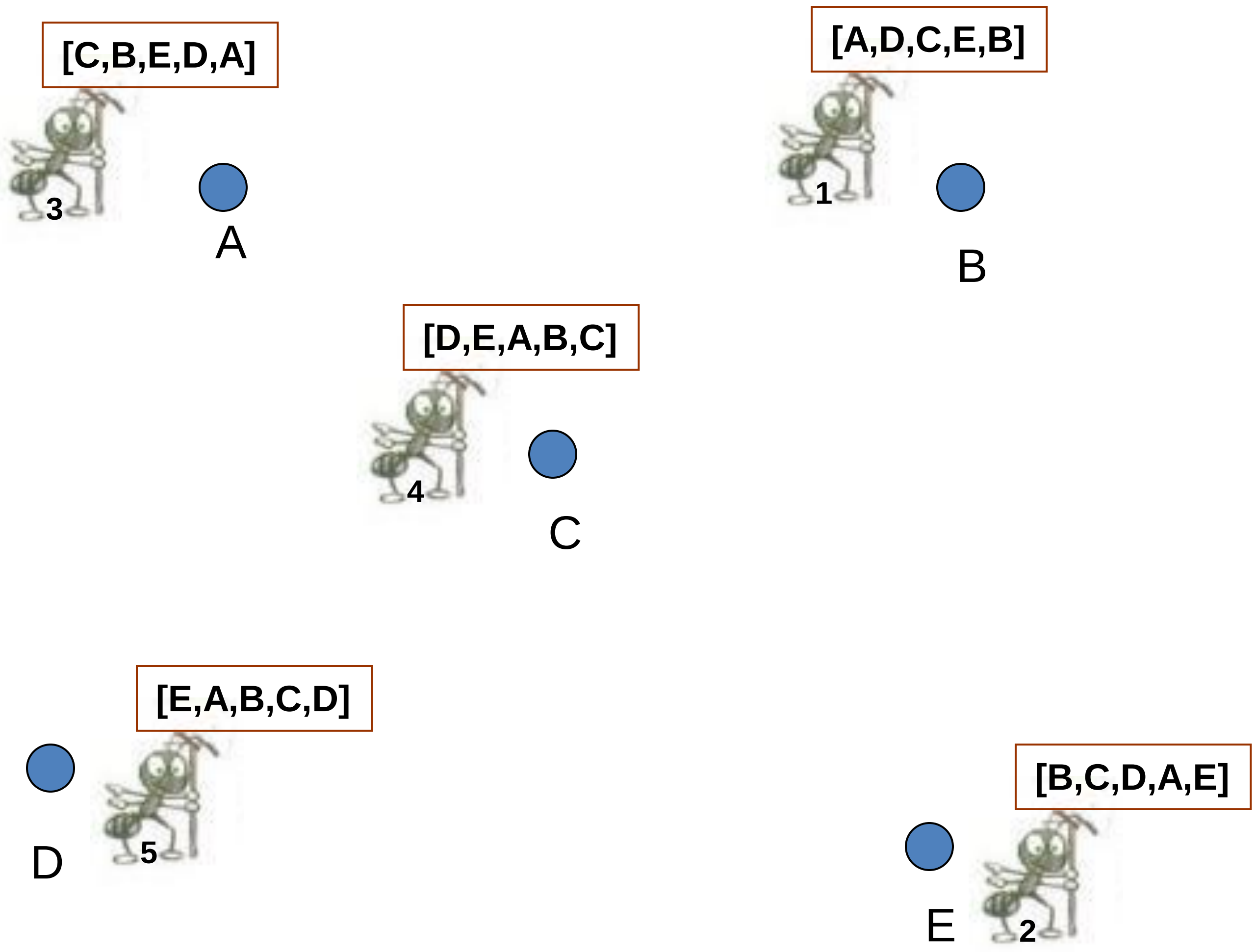
Iteration 3



Iteration 4



Iteration 5



Pheromone Update

[A,D,C,E,B] $C^1 = 300$

[B,C,D,A,E] $C^2 = 450$

[C,B,E,D,A] $C^3 = 260$

[D,E,A,B,C] $C^4 = 280$

[E,A,B,C,D] $C^5 = 420$

$$\Delta\tau_{i,j}^{k} = \begin{cases} \dfrac{1}{C^k} & \text{if } (i,j) \in \text{tour} \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_{A,B} \leftarrow \tau_{A,B} + \Delta\tau_{A,B}^{1} + \Delta\tau_{A,B}^{2} + \Delta\tau_{A,B}^{3} + \Delta\tau_{A,B}^{4} + \Delta\tau_{A,B}^{5}$$

Ant Colony System (ACS) for TSP

- **Dorigo & Gambardella (1997)** introduced four modifications in AS :

1) Different tour construction rule

2) Different pheromone trail updates
   - Global
   - Local

ACS for TSP

- Tour Construction
  - When located at city i, ant k chooses a next city j by the following rule

$$j = \begin{cases} \arg\max argmax_{l \in N^k{}_i} \{\tau_{ij} \, [\eta_{il}]^\beta\}, & \text{if } q \le q_o \\ J\ , & \text{otherwise} \end{cases}$$

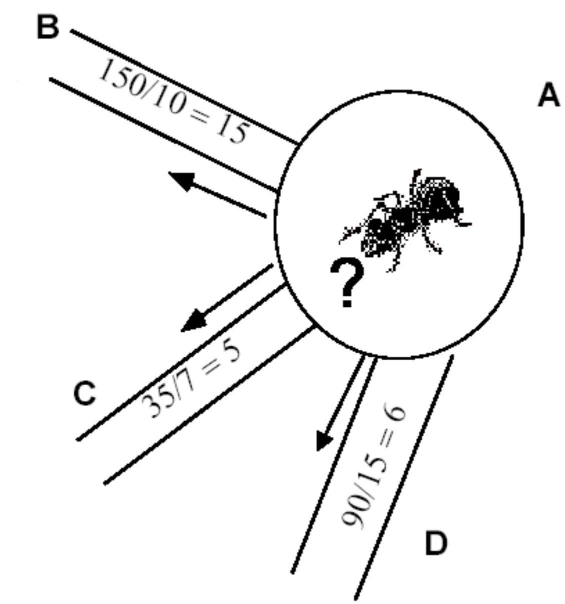
q: random variable uniformly distributed in [0,1]
q_o $\in (0,1]$: parameter
J: random variable selected by the following probability $(\alpha = 0)$

$$p_{ij}^k(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{ik}(t)]^\alpha [\eta_{il}]^\beta} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases}$$

ACS for TSP

- Tour Construction (Example)

$$\tau_{A,B} = 150 \quad \eta_{A,B} = 1/10$$
$$\tau_{A,C} = 35 \quad \eta_{A,C} = 1/7$$
$$\tau_{A,D} = 90 \quad \eta_{A,D} = 1/15$$



If $q \leq q_o$

choose A-B (15)

else

choose

A-B with probability 15/26

A-C with probability 5/26

A-D with probability 6/26

ACS for TSP

- Global pheromone trail update
  - Add and evaporate pheromone only to the best tour since trial began

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\,\Delta\tau^{best}{}_{ij} \qquad \forall (i,j) \in T^{best}$$

$$\Delta\tau^{best}{}_{ij} = {}^{1}\!/_{C^{best}}$$

  - Computational complexity

$$O(n^2) \;\rightarrow\; O(n)$$

ACS for TSP

- Local pheromone update
  - Remove pheromone to the arc (i, j) immediately after having been crossed during tour construction

$$\tau_{ij} \leftarrow (1 - \xi)\tau_{ij} + \xi \, \tau_o$$

$\xi \in (0,1)$, τ_o : initial pheromone

  - Tabu effect
    - The arc recently visited is not desirable for the following ants
    - Increase the exploration of arcs that have not been visited yet

    ⇒ reduce stagnation behavior

ACS for TSP

- Results for a 30 cities instance

	best	average	std.deviation
ACS	420	420.4	1.3
Tabu-search	420	420.6	1.5
Sim. Annealing	422	459.8	25.1

- Results for larger cities (best tour)

	ACO	Gen.Alg.	Evol.Prog.	Sim.Ann.
50 cities	425	428	426	443
75 cities	535	545	542	580
100 cities	21282	21761		

Conclusions

- ACO is a recently proposed metaheuristic approach for solving hard combinatorial optimization problems.
- Artificial ants implement a randomized construction heuristic which makes probabilistic decisions.
- ACO can find best solutions on smaller problems
- On larger problems converged to good solutions –but not the best
- In ACO Local search is extremely important to obtain good results.