Mobile Robot Kinematics

Wheeled Types

1. Conventional wheels
   1) Powered fixed wheel
      - Driven by motor mounted on a fixed position
      - Axis of rotation is fixed w.r.t the platforms coordinate
   2) Castor wheel
      - Not powered
      - Rotate freely about an axis perpendicular to its axis of rotation
   3) Powered steering wheel
      - Driven by motor and can be steered

✓ Higher load capacity
✓ Higher tolerance for ground irregularities
✓ Not omnidirectional (≒ Nonholonomic)

Figure 1.16 Conventional wheels (A) fixed wheel, (B) castor wheel, (C) powered steering wheel without any offset, and (D) power steering wheel with longitudinal offset.

Wheel Types

2. Special wheels
    1) Universal wheel
    2) Mecanum wheel
    3) Ball wheel

✓ Greater maneuvarability

Figure 1.17 Three designs of universal wheel.

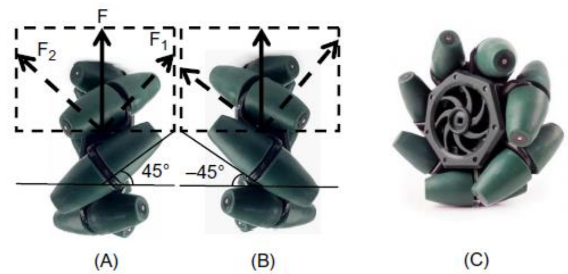


Figure 1.18 (A) Mecanum wheel with $\alpha = 45°$ (left wheel), (B) mecanum wheel with $\alpha = -45°$ (right wheel), and (C) an actual mecanum wheel.

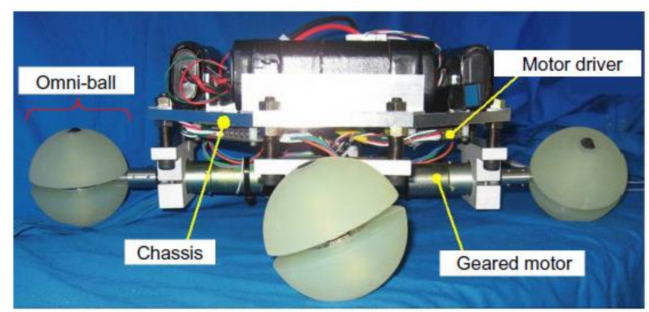


Figure 1.19 A practical implementation of a ball wheel.

Drive Types

- Drives of WMR (wheeled mobile robot)
  - Differential drive
  - Tricycle
  - Omnidirectional
  - Syncro drive
  - Ackerman steering
  - Skid steering

Drive Types

- Differential drive
  - 2 powered fixed wheels + 1 or 2 castor wheels

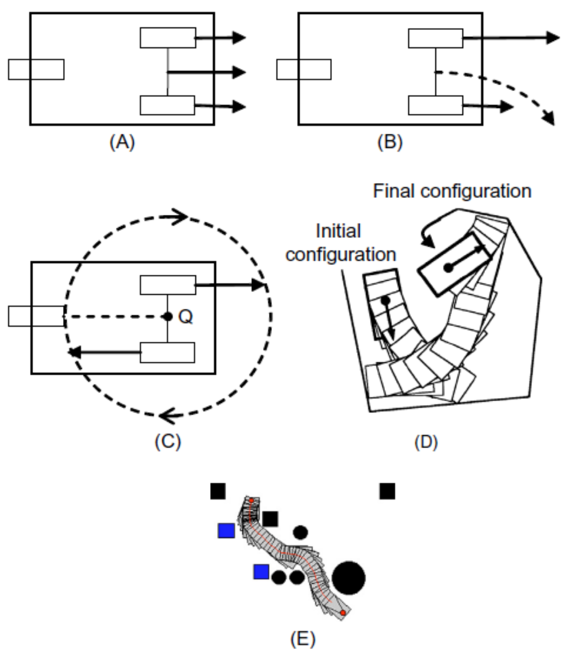
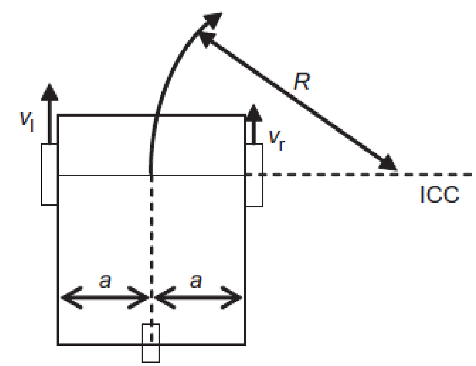


Figure 1.20 Locomotion possibilities of differential drive. (A) Straight path, (B) curved path, (C) circular path, (D) obstacle-free maneuvering to go from an initial to a final pose, and (E) maneuvering to go from an initial to a final pose while avoiding obstacles.

ICC (instantaneous center of curvature)

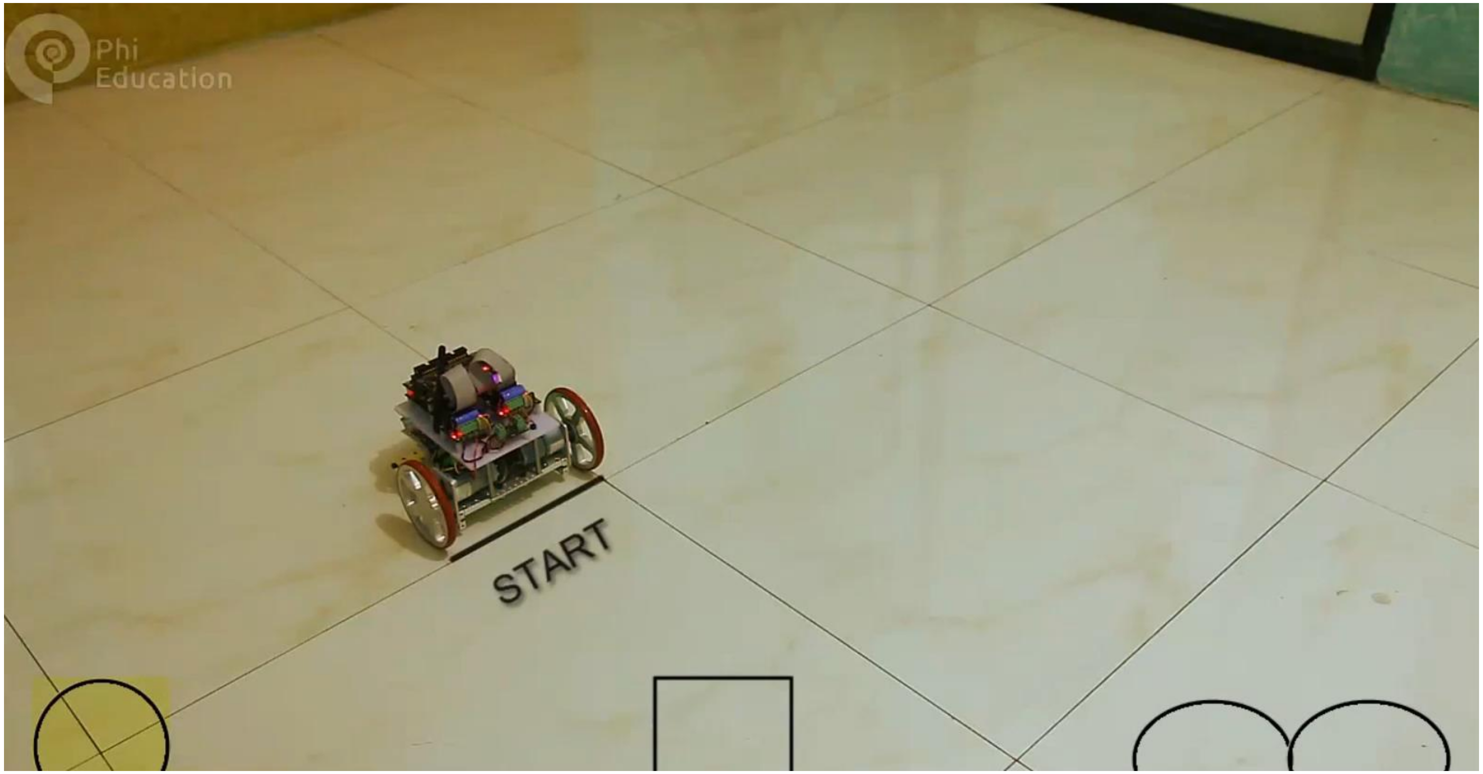


$$v_r = (R-a)\dot{\theta}\,, v_l = (R+a)\dot{\theta}$$

⇨ $$\frac{v_r}{R-a} = \frac{v_l}{R+a}$$

⇨ $$R = a(v_l + v_r)/(v_l - v_r), \quad v_l \geq v_r$$

$$v_r = v_l : R = \infty$$
$$v_r = -v_l : R = 0$$


Phi
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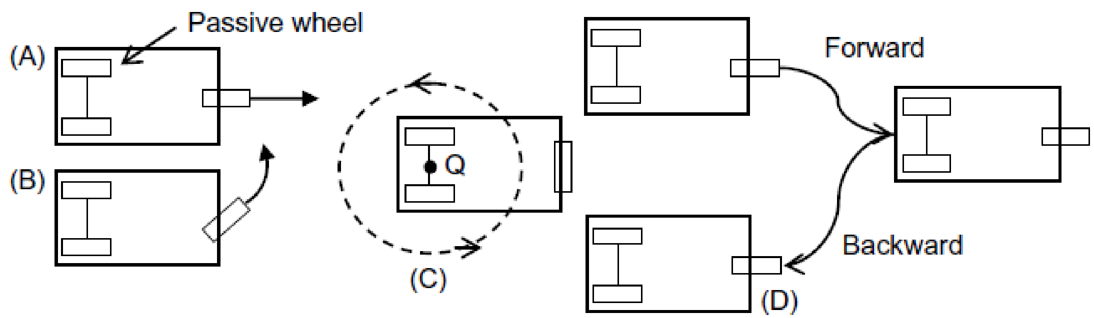
Drive Types

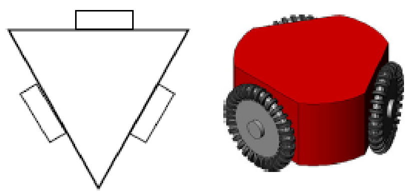
- Tricycle
  - 1 wheel which is both driven and steered + 2 unpowered fixed wheels

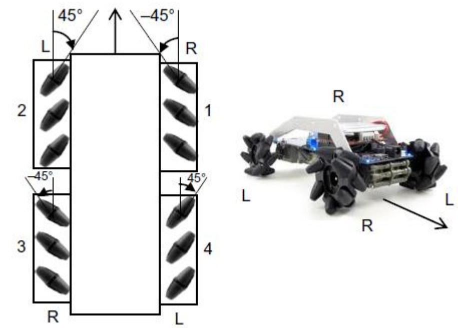


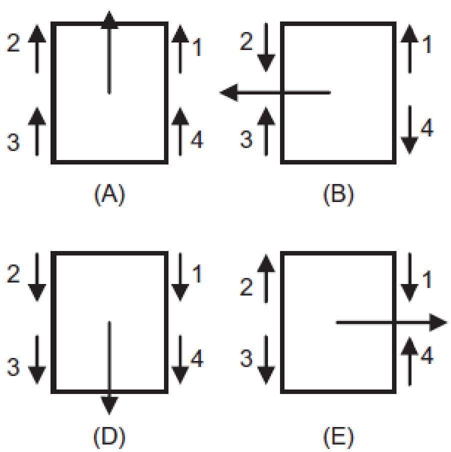
(E)

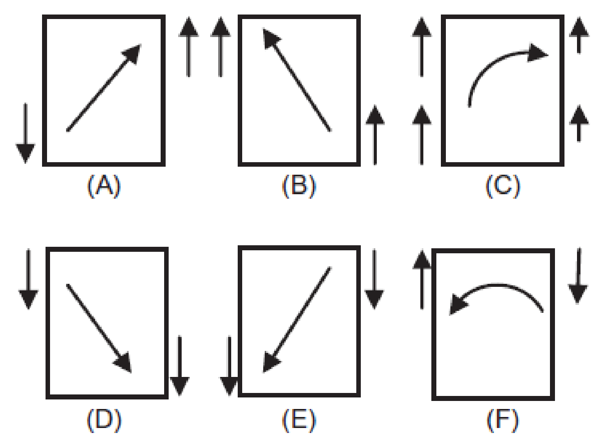
Drive Types

- Omnidirectional
  - 3, 4 or more omnidirectional wheels (universal/mechanum)







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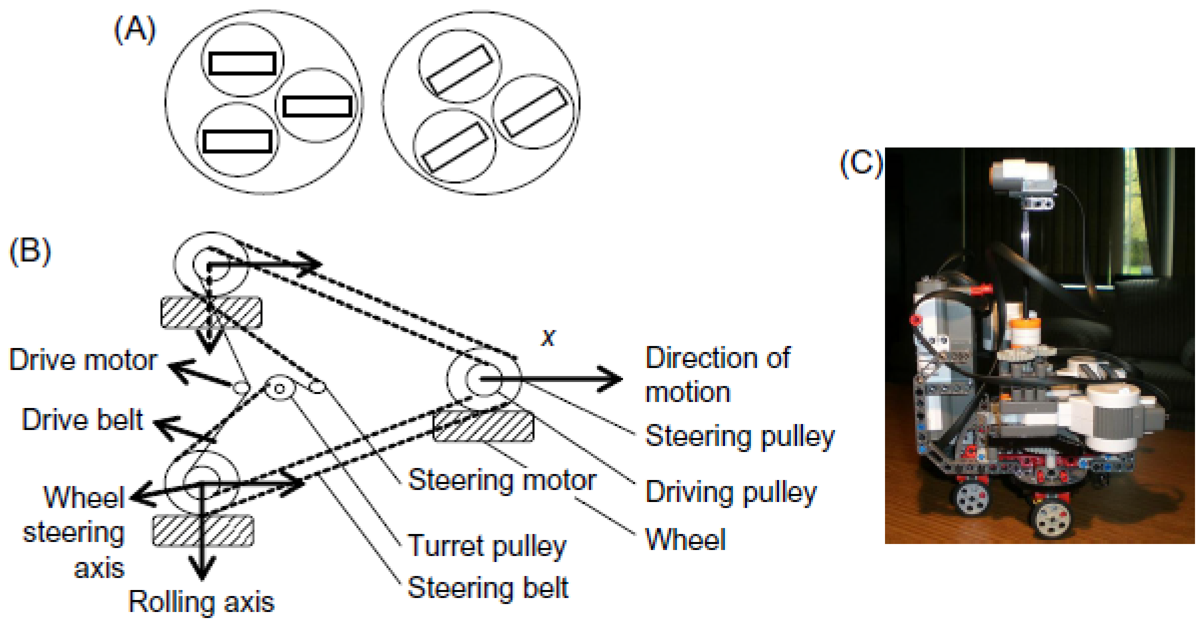
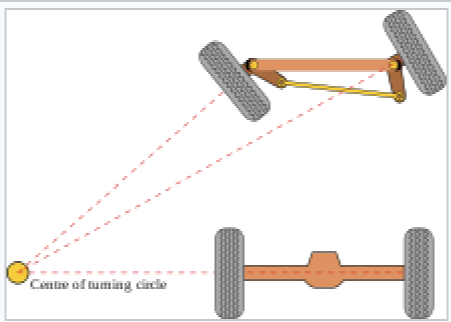
Drive Types

- Synchro drive
  - 3 or more wheels that are mechanically coupled
  - Steering synchronization
  - 1 drive motor & 1 steering motor

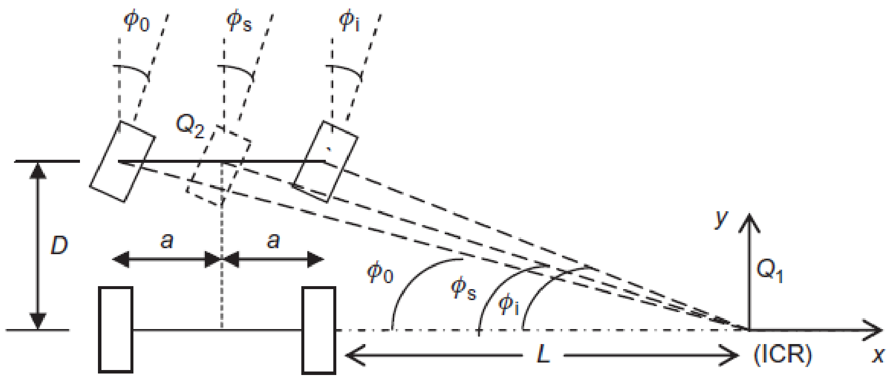


Drive Types

- Ackerman steering
  - Standard steering used in cars
  - 2 combined driven rear wheels + 2 combined steered front wheels
  - Rotation axes of wheels intersect at the same ICR (instantaneous center of rotation)



Ackermann geometry

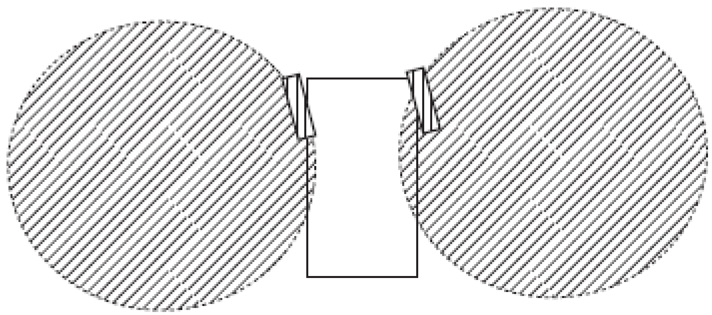


$$ctg\phi_s = (a+L)/D, \quad ctg\phi_o = (2a+L)/D, \quad ctg\phi_i = L/D$$

$$\Rightarrow \quad ctg\phi_s = \frac{a}{D} + ctg\phi_i \quad \text{or} \quad ctg\phi_s = ctg\phi_o - \frac{a}{D}$$

Drive Types

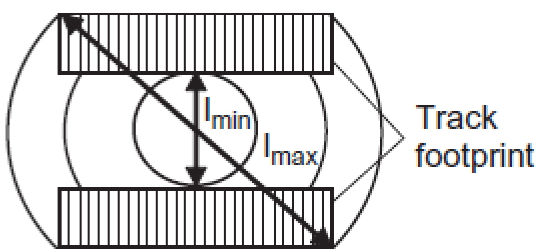
- Ackerman steering
  - For parallel parking, a considerable maneuvering is required



Inaccessible areas of vehicle

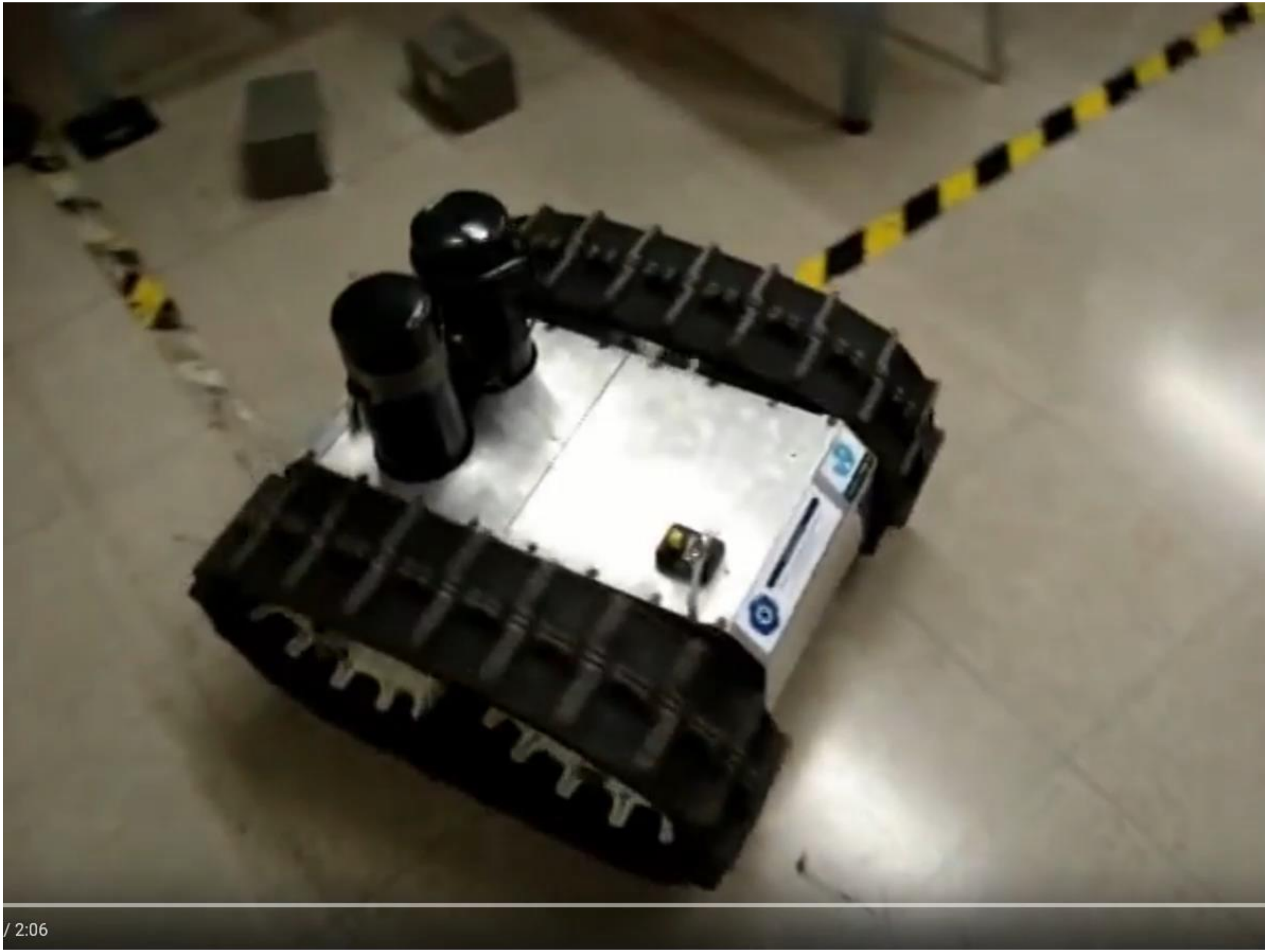
Drive Types

- Skid steering
  - Special implementation of differential drive in track form
  - Increased maneuverability in uneven terrains
  - Higher friction and multiple contact points with terrain



(A)

(B)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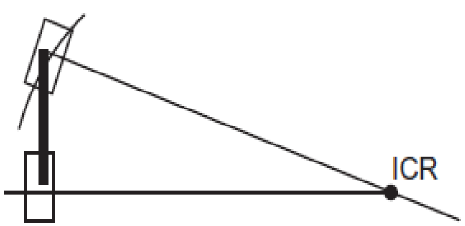
Maneuverability

$M_w = D_m + D_s$ M_w : maneuverability, D_m: degree of mobility, D_m: degree of steerability

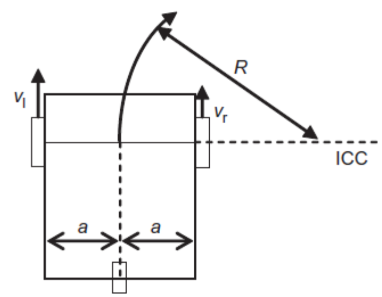
- Degree of mobility (D_m)

  $D_m = 3 - N_c$ N_c : number of independent constraints

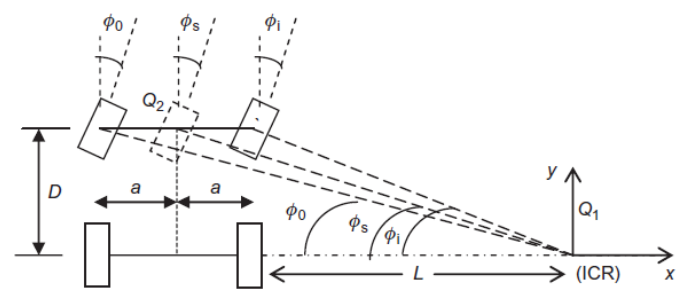
  - Zero motion line
    - line to ICC or ICR along the axis of rotation
    - cannot move the this direction



$N_c = 2$



$N_c = 1$



$N_c = 2$

* $N_c = 0$ for omnidirectional

Maneuverability

- Degree of steerability (D_s)
  - Number of independently controllable steering angles
  - $0 \leq D_s \leq 2$

Table 1.1 Degree of Mobility and Steerability (D_m, D_s) of Typical WMRs

Configuration	D_m	D_s	M_w	Notation
Bicycle	1	1	2	(1,1)
Differential drive	2	0	2	(2,0)
Synchro drive	1	1	2	(1,1)
Tricycle	1	1	2	(1,1)
Ackerman steer	1	1	2	(1,1)
Two-steer	1	2	3	(1,2)
Omni-steer	2	1	3	(2,1)
Omnidirectional	3	0	3	(3,0)

Degrees of Freedom

- DOF (degrees of freedom) vs. DDOF (differential degrees of freedom)

$$\text{DDOF} \le M_{\text{w}} \le \text{DOF}$$

  - DDOF (= D_m) : number of independent velocities
  - DOF : ability to achieve any pose (x, y, ϕ) on the plane

(ex)

Bicycle : DOF =3, DDOF=1

Omnidirectional : DOF = 3, DDOF=3

Wheel Drive Configurations

- One traction wheel in the back, one steering wheel in the front (bicycle, motor cycle)
- Two-wheel differential drive with the center of mass, below the wheels' axis (a balance controller is needed)
- Two-wheel differential drive centered with an omni wheel for stability (Nomad Scout robot)
- Three-wheel differential drive with an unpowered omni wheel, rear or front driven (typical indoor WMRs)
- Two connected powered wheels (differential) in the back, one free turning wheel in front
- One steered and driven wheel in front two free wheels in the back (e.g., Neptune)

- Three omnidirectional wheels (universal)
- Three synchronous powered and steered wheels (synchronous drive)
- Car-like WMR (rear-wheel driven)
- Car-like WMR (front-wheel driven)
- Four-wheel drive, four-wheel steering (Hyperion)
- Differential wheel drive in the back two omni wheels in front
- Four Swedish omnidirectional wheels ($\alpha \neq 90^\circ$) (Uranus)
- Four motorized and steered castor wheels (Nomad XR4000)
- Multi wheel walking drive (rovers, climbing robots).

Figure 1.32 KIVA autonomous mobile robots. (A) Pallet and case handling, (B) Order fulfillment.
Source: (A) www.kivasystems.com/solutions/picking/pick-from-pallets.
(B) www.kivasystems.com/about-us-the-kiva-approach.

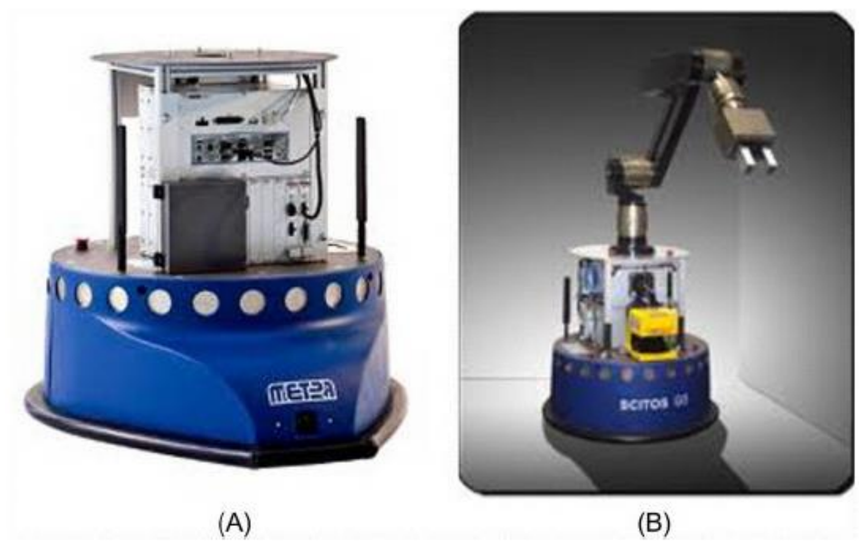


Figure 1.33 (A) SCITOS mobile general platform (SCITOS G5). (B) Robotic manipulator mounted on SCITOS G5.

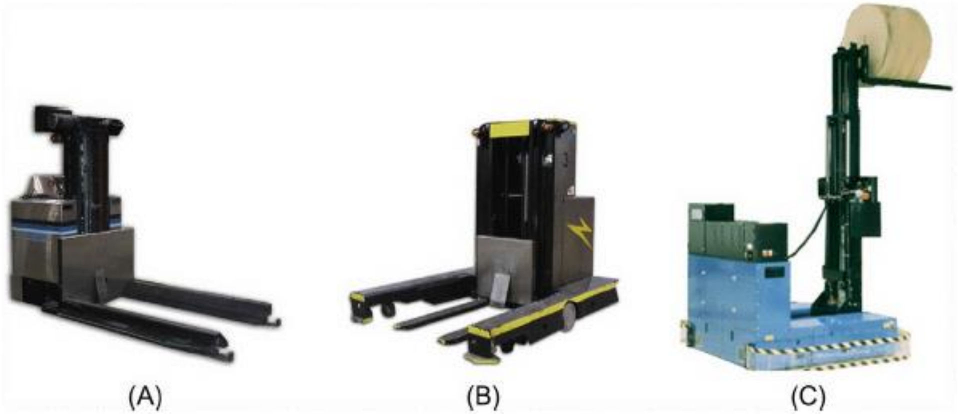


Figure 1.34 CORECON automated guided mobile robots (AGVs). (A) Horizontal roll handling, (B) low lift rear loader, and (C) high lift side loader.
Source: www.coreconagvs.com/products.

Figure 1.36 Mobile robot platform with spring-loaded castors for physical interaction with humans.
Source: http://robot.kaist.ac.kr/paper/view.php?n = 318.

Figure 1.39 The NASA nBot (two-wheel balancing robot).
Source: http://www.geology.smu.edu/~dpa-www/robot/nbot/nobot2/nb12.jpg.

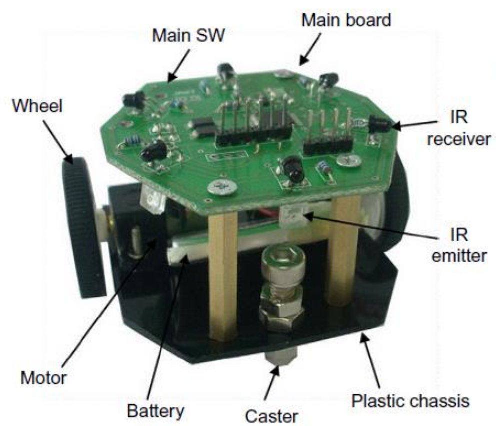


Figure 1.42 The AMiR swarm robot.
Source: http://www.swarmrobotic.com/Robot.htm.

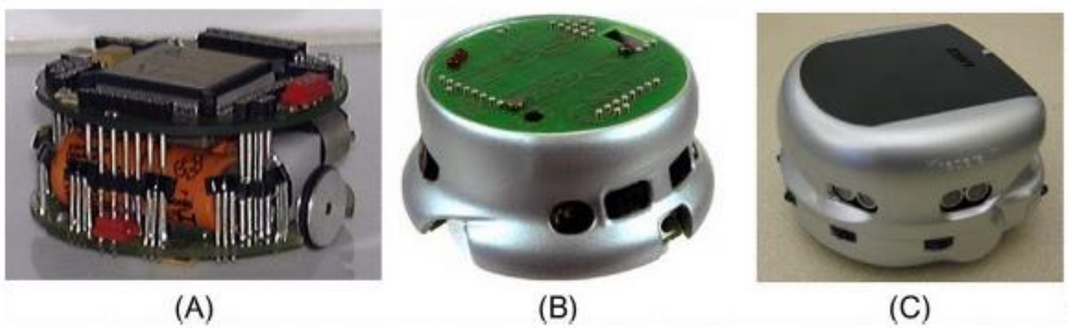


Figure 1.43 The evolution of Khepera WMR. (A) Original version, (B) Version K-II, (C) Version K-III.

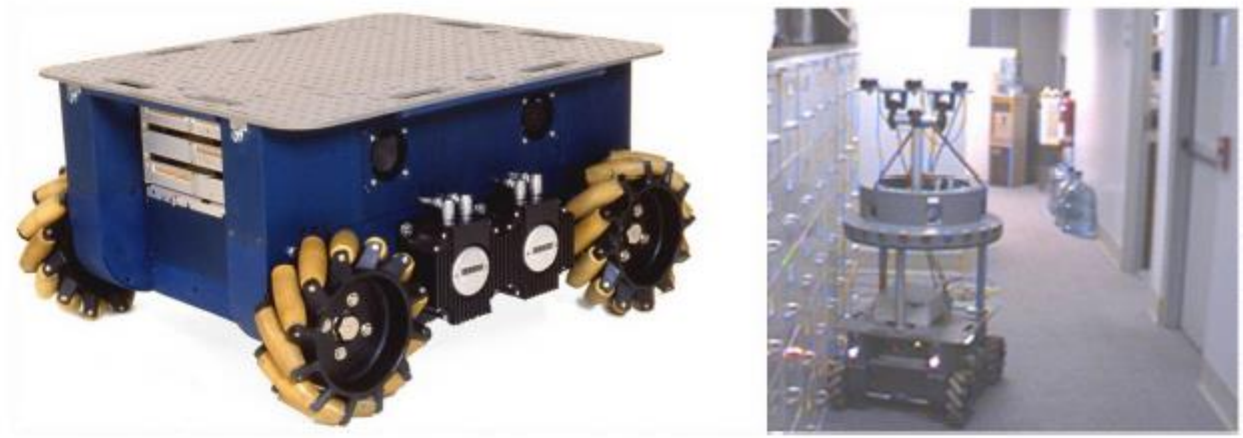
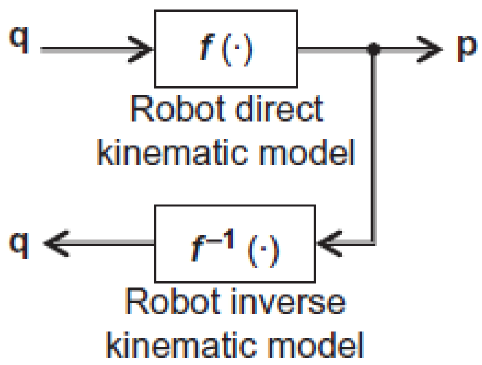


Figure 1.44 "Uranus" four-wheel omnidirectional robot.

Direct and Inverse Kinematics

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

$q_1, q_2, \cdots, q_n$: joint space coordinates (joint space variables)
$x_1, x_2, \cdots, x_n$: task space coordinates (Cartesian space variables)



- Direct (forward) kinematics

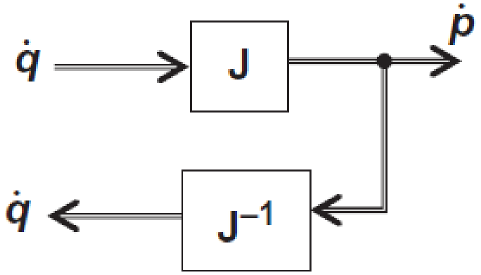
$$\mathbf{p} = \mathbf{f}(\mathbf{q}), \quad \mathbf{f}(\mathbf{q}) = \begin{bmatrix} f_1(\mathbf{q}) \\ f_2(\mathbf{q}) \\ \vdots \\ f_m(\mathbf{q}) \end{bmatrix}$$

- Inverse kinematics

$$\mathbf{q} = \mathbf{f}^{-1}(\mathbf{p})$$

Direct and Inverse Kinematics

- Differential Kinematics



$$\dot{\mathbf{q}} = [\dot{q}_1, \ldots, \dot{q}_n]^T$$

$$\dot{\mathbf{p}} = [\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_m]^T$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \cdots & \frac{\partial x_1}{\partial q_n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial x_m}{\partial q_1} & \frac{\partial x_m}{\partial q_2} & \cdots & \frac{\partial x_m}{\partial q_n} \end{bmatrix} = [J_{ij}]$$

Jacobian matrix (m x n)

$$\dot{\mathbf{p}} = \mathbf{J}\dot{\mathbf{q}}$$

$$\dot{\mathbf{q}} = \mathbf{J}^{-1}\dot{\mathbf{p}}$$ if m = n

Direct and Inverse Kinematics

- Generalized inverse $(m \neq n)$

1) $m > n$ (overspecified)

$$\mathbf{J}^{\dagger} = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T$$

if rank(J) = n

2) $m < n$ (underspecified)

$$\mathbf{J}^{\dagger} = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$$

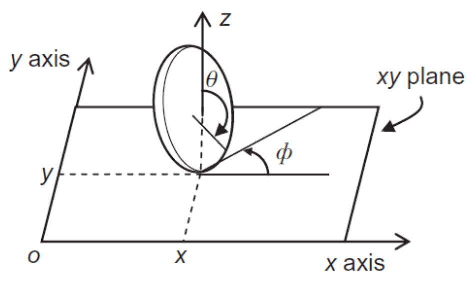
if rank(J) = m

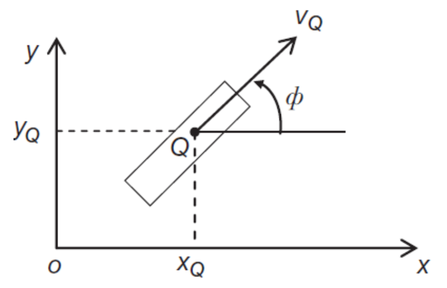
Nonholonomic WMR

- Unicycle
- Differential drive WMR
- Tricycle WMR
- Car-like WMR

Unicycle

- Geometry





$$\mathbf{p} = [x_Q, y_Q, \phi]^T$$

$$\dot{\mathbf{q}} = [v_1, v_2]^T$$

$$v_1 = v_Q$$

$$v_2 = v_\phi$$

- Kinematics

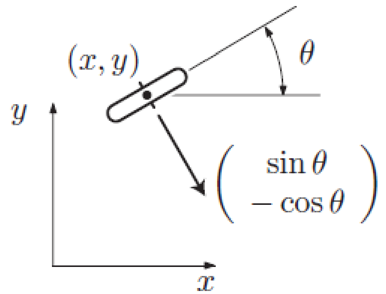
$$\dot{x}_Q = v_Q \cos\phi, \quad \dot{y}_Q = v_Q \sin\phi, \quad \dot{\phi} = v_\phi$$

$$\dot{\mathbf{p}} = \mathbf{J}\dot{\mathbf{q}} \qquad \mathbf{J} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix}$$

nonholonomic constraint $\quad -\dot{x}_Q \sin\phi + \dot{y}_Q \cos\phi = 0$

Unicycle

- Nonholonomic constraint



$$\dot{\mathbf{q}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

Kinematic constraint : Cannot move sideways

$$\dot{x}(t)\ \sin\theta(t) - \dot{y}(t)\cos\theta(t) = 0$$

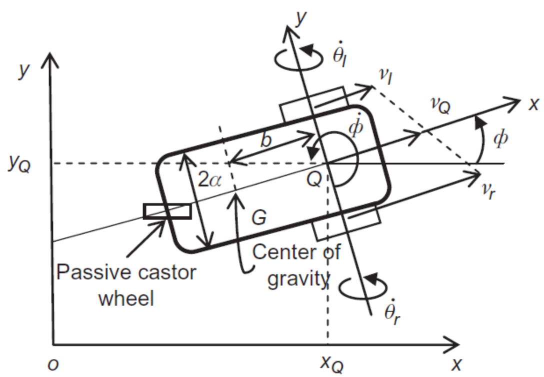
$$f(\mathbf{q}, \dot{\mathbf{q}}, t) = 0$$

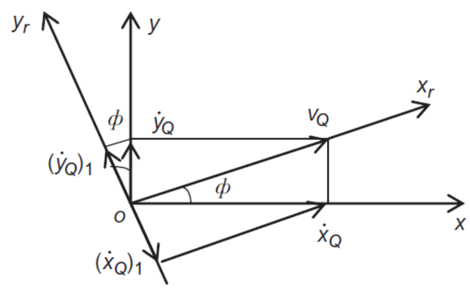
→ Not integrable

→ Nonholonomic constraint

Differential Drive WMR

- Geometry





$$\dot{\mathbf{p}} = \begin{bmatrix} \dot{x}_Q \\ \dot{y}_Q \\ \dot{\phi} \end{bmatrix}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_1 \end{bmatrix}$$

$v_r = r\dot{\theta}_r \qquad v_1 = r\dot{\theta}_1$ (nonslippage assumption)

$$v_r = v_Q + a\dot{\phi}, \quad v_1 = v_Q - a\dot{\phi} \quad \rightarrow \quad v_Q = \frac{1}{2}(v_r + v_1), \quad 2a\dot{\phi} = v_r - v_1$$

Since $\dot{x}_Q = v_Q \cos\phi, \quad \dot{y}_Q = v_Q \sin\phi$

we have

$$\dot{x}_Q = \frac{r}{2}(\dot{\theta}_r \cos\phi + \dot{\theta}_1 \cos\phi)$$

$$\dot{y}_Q = \frac{r}{2}(\dot{\theta}_r \sin\phi + \dot{\theta}_1 \sin\phi)$$

$$\dot{\phi} = \frac{r}{2a}(\dot{\theta}_r - \dot{\theta}_1)$$

Differential Drive WMR

- Kinematic equation

$$\dot{\mathbf{p}} = \mathbf{J}\dot{\mathbf{q}}$$

$$\mathbf{J} = \begin{bmatrix} (r/2)\cos\phi & (r/2)\cos\phi \\ (r/2)\sin\phi & (r/2)\sin\phi \\ r/2a & -r/2a \end{bmatrix}$$

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}\dot{\mathbf{p}}$$

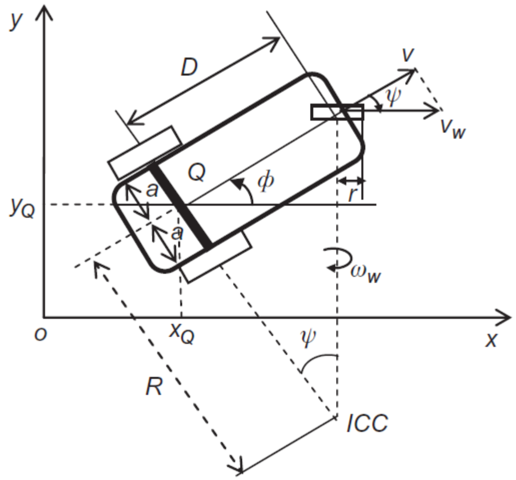
$$\mathbf{J}^{\dagger} = \frac{1}{r}\begin{bmatrix} \cos\phi & \sin\phi & a \\ \cos\phi & \sin\phi & -a \end{bmatrix}$$

- Nonholonomic constraint

$$\mathbf{M}\dot{\mathbf{p}} = 0, \quad \mathbf{M} = \begin{bmatrix} -\sin\phi & \cos\phi & 0 \end{bmatrix}$$

Tricycle WMR

- Geometry



$$\mathbf{p} = [x_Q, \quad y_Q, \quad \phi, \quad \psi]^T$$

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{\theta}_w \\ \omega_\psi \end{bmatrix}$$ linear angular velocity / steering angular velocity

Steering wheel velocity: $v_\mathrm{w} = r\dot{\theta}_\mathrm{w}$.
Vehicle velocity: $v = v_\mathrm{w}\cos\psi = r(\cos\psi)\dot{\theta}_\mathrm{w}$
Vehicle orientation velocity: $\dot{\phi} = (1/D)v_\mathrm{w}\sin\psi$
Steering angle velocity: $\dot{\psi} = \omega_\psi$

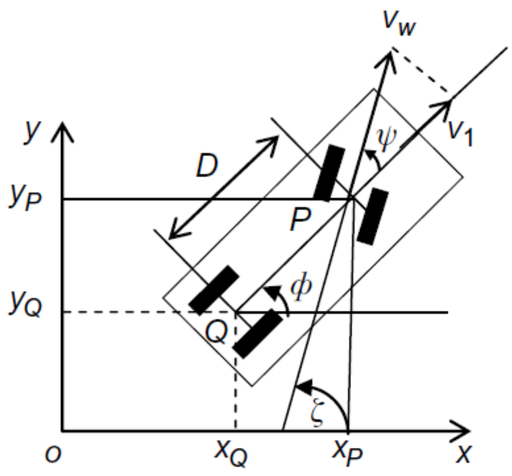
- Kinematic equation

$$\dot{\mathbf{p}} = \begin{bmatrix} \dot{x}_Q \\ \dot{y}_Q \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v\cos\phi \\ v\sin\phi \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} r\cos\psi\cos\phi \\ r\cos\psi\sin\phi \\ (r/D)\sin\psi \\ 0 \end{bmatrix}\dot{\theta}_\mathrm{w} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\dot{\psi} = \boldsymbol{J}\,\dot{\boldsymbol{q}}$$

$$\mathbf{J} = \begin{bmatrix} r\cos\psi\cos\phi & 0 \\ r\cos\psi\sin\phi & 0 \\ (r/D)\sin\psi & 0 \\ 0 & 1 \end{bmatrix}$$

Car-Like WMR

- Geometry



$$\mathbf{p} = [x_Q, y_Q, \phi, \psi]^T$$

$$\dot{\boldsymbol{q}} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$ driving velocity, steering velocity

Rear-wheel driving

$$\dot{x}_Q = v_1 \cos\phi$$
$$\dot{y}_Q = v_1 \sin\phi$$
$$\dot{\phi} = \frac{1}{D} v_w \sin\psi$$
$$= \frac{1}{D} v_1 tg\,\psi$$
$$\dot{\psi} = v_2$$

Car-Like WMR

- Kinematic equation

$$\dot{\mathbf{p}} = \mathbf{J}\mathbf{v}, \quad \mathbf{v} = [v_1, v_2]^T$$

$$\mathbf{J} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ (tg\psi)/D & 0 \\ 0 & 1 \end{bmatrix}$$

- Nonholonomic constraint

$$-\dot{x}_Q \sin\phi + \dot{y}_Q \cos\phi = 0$$ (rear wheel)

$$-\dot{x}_p \sin(\phi + \psi) + \dot{y}_p \cos(\phi + \psi) = 0$$ (front wheel)

$$x_p = x_Q + D\cos\phi, \quad y_p = y_Q + D\sin\phi$$

$$-\dot{x}_Q \sin(\phi + \psi) + \dot{y}_Q \cos(\phi + \psi) + D(\cos\psi)\dot{\phi} = 0$$

$$\Rightarrow \quad \mathbf{M}(\mathbf{p})\dot{\mathbf{p}} = \mathbf{0}$$

$$\mathbf{M}(\mathbf{p}) = \begin{bmatrix} -\sin\phi & \cos\phi & 0 & 0 \\ -\sin(\phi+\psi) & \cos(\phi+\psi) & D\cos\psi & 0 \end{bmatrix}$$